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Stochastic stability of multi-nanobeam systems

Ivan R. Pavlović*, Danilo Karličić, Ratko Pavlović, Goran Janevski, Ivan Ćirić

University of Niš, Faculty of Mechanical Engineering, A. Medvedeva 14, 18000 Niš, Serbia

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ABSTRACT

The paper analyzes a stochastic stability problem of a multi-nanobeam system subjected to compressive axial loading. It is assumed that each pair of nanobeams is simply supported and continuously joined by a viscoelastic layer. Differential equations of nanobeams are given according to Eringen's nonlocal elasticity theory of Helmholtz and bi-Helmholtz type of kernel and Euler–Bernoulli beam theory. Each pair of axial forces consists of a constant part and a time-dependent stochastic function. By using the moment Lyapunov exponent method, regions of almost sure stability of a multi-nanobeam system are obtained in a function of different parameters of the viscoelastic medium, axial loadings and number of nanobeams. Using the regular perturbation method, an approximated analytical solution of the moment Lyapunov exponent is obtained for a single nanobeam subjected to the white noise process, where the results are successfully confirmed with numerical results using the Monte Carlo simulation method. Numerical determination of the moment Lyapunov exponents is further performed for a higher number of nanobeams and different models of wideband processes.

1. Introduction

By using the nonlocal theory of Eringen, nanobeam structures are widely studied by many researchers. Continuum modeling of nanorods, nanobeams and nanoplates has attracted attention due to its simplicity and computational efficiency. Recently, they have been extensively utilized as nano-structure components for nanoelectromechanical and microelectromechanical systems.

Applications of the nonlocal continuum theory to the nanotechnology was initially addressed by Peddieson, Buchanan, and McNitt (2003), who analyzed the static deformations of beam structures based on the simplified nonlocal Eringen's theory (Eringen, 2002). The nonlocal Euler–Bernoulli beam model was used in Nejad and Hadi (2016b), Nejad and Hadi (2016a), Tuna and Kirca (2016) for deriving the partial differential equation of transverse motion of a nanobeam. In work of Tuna and Kirca (2016) the closed-form analytical solutions of original integral model for static bending of Euler Bernoulli and Timoshenko beams, in a simple manner, for different loading and boundary conditions is derived. Nejad and Hadi (2016b) have analyzed the problem of the static bending of Euler–Bernoulli nano-beams made of bi-directional functionally graded material (BDFGM) with small scale effects. Based on Eringen's non-local elasticity theory this authors in Nejad and Hadi (2016a) have analyzed the free vibration of Euler–Bernoulli nano-beams made of the same material as in their previous work (Nejad & Hadi, 2016b). In Reddy (2007) and Reddy and Pang (2008) proposed detailed studies on the application of nonlocal theory to bending, vibration and stability of nanobeam structures. In the study

* Corresponding author. Fax: +381 18 588 244.
E-mail addresses: pivan@masfak.ni.ac.rs, ivanp79@gmail.com (I.R. Pavlović).

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(Reddy, 2007), the author reformulated classical beam theories such as Euler–Bernoulli, Timoshenko, Reddy and Levinson, by using the nonlocal elasticity theory and then applied them in the determination of an analytical solution for bending, vibration and buckling of a simply supported nanobeam. By considering Timoshenko and Euler–Bernoulli beam model, in the paper (Reddy & Pang, 2008) the authors modeled single walled carbon nanotubes (SWCNT) and determined an analytical solution for natural frequency and critical buckling load. In the review paper by Arash and Wang (2012), the authors applied various nonlocal models for analyzing the mechanical behavior of SWCNT and graphene sheets. Ansari, Gholami, and Rouhi (2012) presented the vibration model of SWCNT based on a different gradient elasticity theory and Euler-Bernoulli and Timoshenko beam theories. Excellent agreement between the results obtained by using various gradient elasticity theories and MD simulations was shown. Wang and Varadan (2006) analyzed the effects of the scale-scale on the natural frequencies, compared with local natural frequencies for both single-walled CNT and double-walled CNT. Their results led to the conclusion that the classical continuum models were still valid and convenient for studying vibration responses of long and wide SWCNTs, especially for lower modes. The dynamic behavior of SWCNT embedded in an elastic medium (matrix), investigated by using nonlocal Timoshenko beam theory for both stress gradient (Eringen nonlocal theory) and strain gradient approach, were considered by Wang and Wang (2007). Their results showed a significant dependence of frequencies on the surrounding medium and nonlocal parameter. The static and dynamic behavior of nanobeams was analyzed based on the nonlocal Euler–Bernoulli, Timoshenko, Reddy, Levinson and Aydogdu beam theories and presented in Aydogdu (2009). The influence of the nonlocal effects and length of a nanobeam on the natural frequencies, deflection and critical load was investigated in detail for each considered model.

Recently, the complex models of nanostructure systems composed of multiple nanorods, nanobeams or nanoplates embedded in a certain type of medium have become very interesting for research and design in nano-engineering (Karličić, Murmu, Adhikari, & McCarthy, 2015b). Such systems represent a mechanical model of coupled multi SWCNT as a beam system and graphene nano-sheets as a system of nanoplates, embedded in an elastic or viscoelastic medium. Murmu and Adhikari (2012) investigated a double-nanobeam system based on the nonlocal elasticity and proposed an analytical method for obtaining the natural frequencies of the system under the influence of the initial compressive pre-stressed condition. Kelly and Srinivas (2009) showed a methodology for determination of natural frequencies and mode shapes of the elastically connected axially loaded Euler-Bernoulli beams, based on the classical elasticity theory. Pouresmaeeli, Fazelzadeh, and Ghavanloo (2012) analyzed the nonlocal vibrations of double-orthotropic nanoplates embedded in an elastic medium. They obtained analytical expressions of the nonlocal natural frequencies and investigated the influences of the small scale coefficient, stiffness of the external and internal mediums and aspect ratio on the vibration of the simply-supported double-orthotropic nanoplates. In the work by Wang, Li, and Kishimoto (2011), the thermal effects on the dynamic behavior of the double nanoplates systems were analyzed. They obtained analytical expressions for the natural frequencies, based on the nonlocal elasticity theory and showed the influence of axial stress caused by the thermal effects. The paper by Pradhan and Phadikar (2009) investigated free vibration of multilayered graphene sheets embedded in polymer matrix by analytical methods. In Liew, He, and Kitipornchai (2006) vibration behavior of multi-layered graphene sheets that were embedded in an elastic matrix was considered. The authors obtained a set of explicit formulas to predict the natural frequencies and associated vibration modes, only for the case of double-layered and triple-layered graphene sheets that were embedded in an elastic matrix. Ansari, Arash, and Rouhi (2011) offered a nonlocal continuum model for nano-scale vibration of an embedded multi-layered graphene sheet based on the Reissner–Midlin plate theory. By considering the nonlocal elastic and viscoelastic theory, Karličić, Kozić, and Pavlović (2016), Karličić, Kozić, and Pavlović (2014a), Karličić, Kozić, Murmu, and Adhikari (2015a), Karličić, Murmu, Cajić, Kozić, and Adhikari (2014b) analyzed vibrations and stability behavior of a system composed of m elastic and viscoelastic nanorods, nanobeams and nanoplates embedded in an elastic and viscoelastic medium. Based on the D' Alembert's principle, the system of governing equations of motion was derived and then solved by the trigonometric method. An analytical solution of natural frequencies and critical buckling load was obtained, and the nonlocal effects on the dynamic and stability behavior of systems was shown.

By studying the literature, it can be concluded that the stochastic stability of nanostructure elements is investigated in a small number of papers. In general, dynamic stability of nanobeam-like structures plays a significant role in the design of future nanodevices. For an axially loaded nanobeam, where the load is a time dependent stochastic function, a failure occurring due to dynamic instability under a load may be much smaller than the failure induced by static buckling. These instability conditions usually lead to the failure of nanodevices. By considering the method of maximal Lyapunov exponent and taking into account the effect of the nonlocal elasticity, almost sure asymptotic stability of a beam was investigated by Potapov (2013). According to the direct Lyapunov method and the moment Lyapunov exponent method, dynamic stability of a viscoelastic nanobeam was analyzed in Pavlović, Pavlović, Ćirić, and Karličić (2015a). In this work moment Lyapunov exponents of the viscoelastic nanobeam under real noise excitation were numerically determined using the Monte Carlo simulation method. Based on the Euler–Bernoulli and Timoshenko beam theories, stochastic stability of nanobeams subjected to time dependent random loads were investigated by Tylikowski, (2011b), Tylikowski, (2011a). In order to obtain stability boundaries for two coupled elastic beams, Kozić, Janevski, and Pavlović (2010) and Pavlović, Kozić, and Pavlović (2012) derived approximated analytical solutions for the moment Lyapunov and Lyapunov exponents. The authors also analyzed the influence of different physical effects on the stochastic stability of the system.

Due to the best of authors' knowledge there are no works in the literature having a analysis of stochastic stability problem of a coupled viscoelastic nanobeam performed by using the continuum, numerical or molecular dynamic approach. Understanding the stability behavior of such multiply systems, especially complex multi-nanobeam system could be a key