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Stability of flows heated by friction

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ABSTRACT

Simple flows can be used to measure viscosities. If the viscosity is high enough and depends on temperature, frictional heating presents a technical difficulty. Yet in many viscometric flows formulas predicting the temperature and the velocity distribution are available. The resulting flow may or may not be stable to small perturbations. If it is unstable it will not be seen in an experiment. Our aim is to show that stability depends on how the experiment is controlled, whether by holding a kinematic or a dynamic variable fixed, eg., wall speed or wall stress. In a plane Couette flow, controlled by the wall speed, stability to long wave length perturbations is established for all points of the base curve. In both a rotational Couette flow and a Poiseulle flow stability obtains at least part way along the upper branch.

1. Introduction

Frictional heating may be important in the flow of viscous fluids, the more so, the stronger the dependence of their viscosity on their temperature. At a given stress, as the viscosity decreases, the velocity gradient increases and hence the heat generation increases, being linear in the viscosity, quadratic in the velocity gradient.

The heat generated must be conducted to the walls, thus the more distant a point is from the wall, the higher the temperature and the lower the viscosity. If this increases the heat generation we have a positive feed back leading to curves like that sketched in Fig. 1 where a characteristic temperature rise is plotted versus a measure of the imposed shearing. The symbols L, N and U denote the lower branch, the nose and the upper branch. Formulas are available for these curves in plane and circular geometries, cf., Joseph (1964); Kearsley (1963).

Our interest is in the stability of these flows and our aim is to present some simple results showing that stability depends on how the flows are controlled.

In the case of plane Couette flow many computational results are available if the flow is controlled by holding the wall speed fixed, cf., Sukanek, Goldstein, and Laurence (1973) and Yueh and Weng (1996). But, without carrying out a computation, it is possible to draw some conclusions about the stability of frictionally heated flows in the case of long wave length perturbations, perturbations which adjust the flow speed on the flow cross section but do not include secondary motions. Oron, Davis, and Bankoff (1997) present a guide to long wave length approximations. Due to the fact that long wave length perturbations make the stabilizing effect of heat loss depend entirely on conduction, they ought to be dangerous.

Joseph (1965) finds the lower branch stable, the nose critical and the upper branch unstable to long wave length perturbations holding the stress at the moving wall fixed. Johns and Narayanan (1997) find that the nose is stable and stability persists someway along the upper branch if the wall speed, in place of the wall stress, is held fixed on perturbation. Again,

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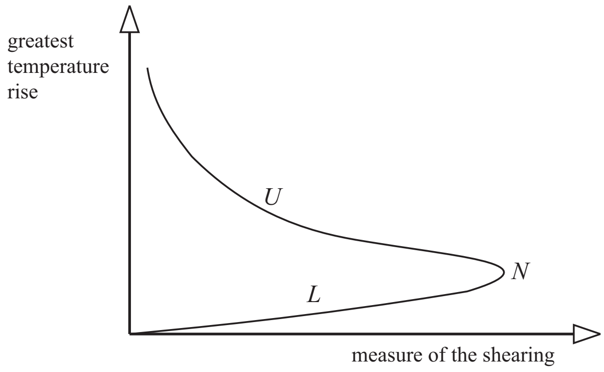

Fig. 1. The Base Solution Curve.

both a Couette flow and a long wave length result. They also show that upon moving along the base curve (lower branch $\rightarrow$ nose $\rightarrow$ upper branch) eigenvalues, denoted subsequently by λ^2 and μ^2, are always decreasing. This result is not limited to plane Couette flow and we use it herein.

We extend this Couette flow result to the entire upper branch and prove stability on the upper branch, at least near the nose, in two other simple flows.

Given a flow, we introduce a long wave length perturbation, and seek points along the base curve where its growth rate vanishes. Thus we wish to know if the perturbation problem at zero growth rate has a solution other than zero. And to answer this question we present two eigenvalue problems which correspond to two ways of running an experiment.

If the wall stress is the input, the fundamental eigenvalue is positive. If the wall speed is the input, the fundamental eigenvalue is negative, thus making an important distinction between these two ways the flow can be controlled.

2. Scaling

We assume the fluidity, the reciprocal of the viscosity, is an exponential function of temperature, viz.,

$$f(T) = f(T_{\text{wall}})e^{\alpha(T - T_{\text{wall}})}, \quad \alpha > 0$$

Then, denoting by T the temperature above the wall temperature scaled by $\dfrac{1}{\alpha}$ and scaling the viscosity by its wall value, we have for the scaled fluidity

$$f(T) = e^T = \frac{df(T)}{dT}$$

We scale all lengths by a characteristic length and the stress components by the square root of the product of the viscosity at the wall and the thermal conductivity divided by the product of the square of the characteristic length and α.

The velocity components are scaled by the square root of the thermal conductivity divided by the product of the viscosity at the wall and α.

Henceforth all variables are scaled.

3. The model

Our model is the Navier-Stokes model accounting for the temperature dependence of the viscosity, viz.,

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v} \cdot \nabla \vec{v} = -\nabla p + \nabla \cdot \vec{\vec{S}}$$

$$\nabla \cdot \vec{v} = 0$$

$$\vec{\vec{S}} = \mu\, 2\vec{\vec{D}} = \mu\left(\nabla \vec{v} + \nabla \vec{v}^{\,T}\right)$$

and

$$\left(\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T\right) = \nabla^2 T + \mu 2\vec{\vec{D}} : \vec{\vec{D}}$$

Our problems are two dimensional, having one non zero velocity component, i.e., we address the temperature rise in viscometric flows.