International Journal of Engineering Science 109 (2016) 115–164

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

On the mechanics of laminated doubly-curved shells subjected to point and line loads

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ARTICLE INFO

Article history:
Received 14 July 2016
Revised 10 August 2016
Accepted 1 September 2016

Keywords:
Laminated composite doubly-curved shells
Point and line loads
Generalized differential quadrature method
Higher-order equivalent single layer theories

ABSTRACT

It is well-known that the implementation of concentrated forces, such as point and line loads, represents a challenging task, especially from the computational point of view, since a strong discontinuity has to be inserted in the structural model. The present paper aims to solve the static problem of laminated composite doubly-curved shell structures subjected to concentrated loads employing the Generalized Differential Quadrature (GDQ) as numerical tool, according to what has been shown by the authors in their previous works. Its accuracy and reliability features are proven for several grid distributions when the concentrated loads are modeled through the Dirac-delta function. The theoretical framework on which this approach is based is a Unified Formulation, which allows to investigate several Higher-order Shear Deformation Theories (HSDTs). The differential geometry is used to describe accurately the reference surface of various doubly-curved shell structures. The validity of the current approach is shown comparing the GDQ results with the exact and semi-analytical ones available in the literature. A posteriori recovery procedure based on the three-dimensional equilibrium equations for a shell structure is introduced to compute the through-the-thickness variation of strain, stress and displacement components by means of the GDQ method.

1. Introduction

In the last decades, composite materials have considerably affected the way of manufacturing and analyzing several kinds of engineering structures employed in the automotive, aerospace, biomedical, architectural, mechanical, and civil sectors. Their increasing use is proven by the huge number of research papers published in the last years. All these aspects are discussed in depth in the book by Reddy (Reddy, 2004).

One of the most exploited types of composite materials is represented by fibrous composites. For the sake of completeness, it should be recalled that a fiber-reinforced medium consists of several fibers embedded in a matrix according to prearranged paths. Due to this micromechanical configuration, high level performance can be reached if compared with an isotropic material. It is clear that the mechanical properties of these composites depend on the orientation of the reinforcing fibers and their volume fraction, as well as their strength and stiffness (Tornabene, Fantuzzi, Bacciocchi, & Viola, 2016c,d). The remarkable mechanical properties of fibrous composites can be further improved by combining together different fiber-reinforced layers in order to obtain the so-called laminated composite. The layers can be arranged in several manners, according to the lamination scheme, or stacking sequence. The way these composites are assembled, as well as the

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http://dx.doi.org/10.1016/j.ijengsci.2016.09.001
0020-7225/

orientation and the mechanical properties of each single fiber-reinforced layer, must be considered to obtain a peculiar structural response (Brischetto, 2014; Brischetto, & Torre, 2014; Eftekhari, & Jafari, 2014; Groh, & Weaver, 2015a,b; Jin, Ye, Wang, & Miao, 2016; Kulikov, Mamontov, Plotnikova, & Mamontov, 2016; Le, & Yi, 2016; Librescu, & Reddy, 1989; Mohammad-Abadi, & Daneshmehr, 2015; Mohammad-Abadi, Daneshmehr, & Homayounfard, 2015; Mohazzab, & Dozio, 2015; Piskunov, Verijenko, Adali, & Summers, 1993; Vescovini, & Dozio, 2016; Wu, & Hung, 1999; Yifeng, Wenzheng, Yu, Wenwen, & Lichao, 2015).

It is evident that laminated composites are heterogeneous mediums. As a consequence, the well-known phenomenon of delamination can happen due to the high values of shear stresses that can be registered among the laminae. For this purpose, the class of granular composites (also known as Functionally Graded Materials or FGMs), has been developed to avoid this problem (Alibeigloo, 2016; Behravan Rad, & Shariyat, 2015; Fazzolari, 2014, 2015, 2016a,b; Guo, Chen, & Pan, 2016; Jooybar, Malekzadeh, Fiouz, & Vaghefi, 2016; Kim, 2015; Liu, Cheng, & Liu, 2015; Mantari, 2015; Mercan, Demir, & Civalek, 2016; Quan, Tran, Tuan, & Duc, 2015; Shafiei, Kazemi, Safi, & Ghadiri, 2016; Sofiyev, 2015, 2016; Sofiyev, & Kuruoglu, 2015, 2016). Recently, due to the advancements in the manufacturing process, innovative materials such as piezoelectric ones can be inserted in the lamination scheme as distributed sensors and actuators for an active control on the structural conditions, as highlighted in the work by Ray and Reddy (Ray, & Reddy, 2005). This idea has contributed to the development of the so-called smart structures (Brischetto, & Carrera, 2013; Cheng, Lim, & Kitipornchai, 1999; Dehghan, Nejad, & Moosaie, 2016; Hadjiloizi, Kalamkarov, Metti, & Georgiades, 2014b,a). Another way to enhance the structural response of composite materials that is attracting recently the interest of many researchers consists in inserting carbon nanoparticles in a polymeric matrix as reinforcing phase (Brischetto, Tornabene, Fantuzzi, & Bacciocchi, 2015; Kamarian, Salim, Dimitri, & Tornabene, 2016; Tornabene, Fantuzzi, Bacciocchi, & Viola, 2016). As a consequence, more and more complex constitutive laws must be introduced to take into account these mechanical configurations. Simultaneously, even the fundamental assumptions and the kinematic model must be adequate to capture the effective structural response. For these purposes, several Higher-order Shear Deformation Theories (HSDTs) have been developed recently by many researchers. One of the most efficient approaches to deal with these HSDTs can be found in the works by Carrera (Carrera, 2002, 2003, 2004), in which a Unified Formulation was presented. Its efficacy is given by its generality. In other words, many different higher-order theories can be developed and investigated by means of only one formulation. In addition, the order of kinematic expansion which characterizes the enrichment of the displacement field can be chosen arbitrarily.

In the current paper, the aforementioned theoretical formulation is employed to investigate the static behavior of laminated structures subjected to concentrated forces, starting from the preliminary results and considerations presented by the authors in their previous work (Tornabene, Fantuzzi, Bacciocchi, & Viola, 2015c). In general, concentrated loads represent a strong discontinuity for those researchers who aim to find a solution to these kinds of structural problems employing a numerical approach. At this point, a brief and partially complete literature review concerning concentrated load problems is presented. The classical laminated plate theory was employed by Becker in his work (Becker, 1995) to study the static behavior of infinitely extended unsymmetrical laminates subjected to concentrated forces and moments, developing a complex potential approach. Kim and Swanson (Kim, & Swanson, 2001) presented some analyses to design sandwich structures in order to bear concentrated loading, highlighting the typical failure modes that characterize these peculiar laminates. The issue of composite laminates with elliptic elastic inclusions subjected to concentrated forces and moments was considered in the paper by Hwu and Tan (Hwu, & Tan, 2007), basing on the Kirchhoff's assumptions for thin plate. The Boundary Element Method was used by Tsamasphyros et al. (Tsamasphyros, Theotokoglou, & Filopoulos, 2013) to solve some problems of structures loaded by concentrated forces and moments. Abali et al. (Abali et al., 2014) combined an analytical model with computational means to study the three-dimensional elastic deformations of isotropic functionally graded plates subjected to point loads, putting in evidence some limitations typical of finite element approaches. An experimental analysis was carried out by Nikopour and Selvadurai (Nikopour & Selvadurai, 2014) to investigate the flexural behavior of fiber-reinforced rectangular plates with different boundary conditions. The results related to the application of a central load were compared to the numerical solutions obtained through a finite element model. The same kind of analysis was realized by Cernescu and Romanoff in their paper (Cernescu, & Romanoff, 2015) to study the bending deflections of sandwich beams in three-point bending. Finally, Eftekhari introduced the Dirac-delta function in order to apply a concentrated load combining differential quadrature and integral quadrature methods (Eftekhari, 2015). In particular, the accuracy and the reliability of the present approach were both proven by the analysis of beams and plates under concentrated loads. It can be easily noticed that only flat structures have been analyzed in these papers. The work by Reddy (Reddy, 1984) represents one of the few examples related to laminated shells subjected to central point loads. In his paper, several benchmark solutions were obtained using a semi-analytical approach, starting from the Sanders theory for doubly-curved shallow shells. A typical problem of shell structures subjected to concentrated forces is represented by the problem of the pinched cylinders (Beirão da Veiga, 2005; Jones, 1998b,a; Morley, 1960; Ting, & Yuan, 1958; Yuan, 1946; Yuan, & Ting, 1957; Zhang, 1991). For the sake of completeness, it should be recalled that the pinched problem consists in a structure subjected to concentrated, equal and opposite loads, applied on the diameter. In the works by Yuan (Yuan, 1946) and Yuan and Ting (Yuan, & Ting, 1957), the radial deflections of infinitely long and finite-length cylinders were computed, respectively. The loading function was modeled by a Fourier integral in the longitudinal direction and by a Fourier series along the circumferential direction. The radial deformations of thin-walled pinched circular cylinder with simply supported and free edges were evaluated by Ting and Yuan (Ting, & Yuan, 1958). Morley (Morley, 1960) obtained an approximate solution of the pinched cylinder problem solving a simplified eighth order equation. A set of parametric studies concerning directional properties was developed by Zhang in his paper