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Discrete model of hydraulic fracture crack propagation in homogeneous isotropic elastic media

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ABSTRACT

An infinite homogeneous and isotropic elastic medium with a penny shape crack is considered. The crack is subjected to the pressure of fluid injected in the crack center. Description of the crack growth is based on the lubrication equation (balance of the injected fluid and the crack volume), equation for crack opening caused by fluid pressure on the crack surface, the Poiseullie equation related local fluid flux with the crack opening and pressure gradient, and classical criterion of crack propagation of linear fracture mechanics. The crack growth is simulated by a discrete process consisting of three basic stages: increasing the crack volume by a constant crack size, crack jump to a new size defined by the fracture criterion, and filling the appeared crack volume by the fluid. It is shown that the model results a reasonable dependence of the crack size on the time as well as the pressure distribution of fluid on the crack surface. Comparisons with the solutions of hydraulic fracture problems existing in the literature are presented.

1. Introduction

Because of importance for gas and petroleum industry, the process of hydraulic fracture crack propagation has been the object of intense theoretical and experimental studies for about sixty years. The number of publications dedicated to analytical and numerical solutions of this problem is huge. There are several books and journal surveys where substantial portion of these publications is mentioned. Publications before 21st century can be found in the books of Valko and Economides (1995) and Economides and Nolte (2000). Most recent advances in the area can be found in the work of Linkov (2012); 2016), Mishuris, Wrobel, and Linkov (2012), and Wrobel and Mishuris (2015) and they have already led to a discussion (see Detournay & Peirce, 2014).

Mathematical equations of the process of hydraulic fracture crack growth were derived at the end of last century and presented in the works of many authors (see, e.g., Savitski & Detournay, 2002). It was shown that the problem is reduced to a system of non-linear integro-differential equations in the region with moving boundary. Analytical solutions of these equations do not exist even in simplest cases, e.g., for a penny shape crack in isotropic and homogeneous elastic media. Many attempts were performed to find analytical solutions at least for specific regimens of the fluid injection, properties of fluid (small and large viscosity), and properties of the medium (small and large fracture toughness). Usually such asymptotic solutions were derived by assumptions that cannot be strictly justified.

Numerical solutions of the system of equations of the hydraulic fracture problem were also considered in a number of works (e.g., Meyer, 1989, Adachi, Siebrits, Peirce, & Desroches, 2007). For the solution, conventional numerical methods

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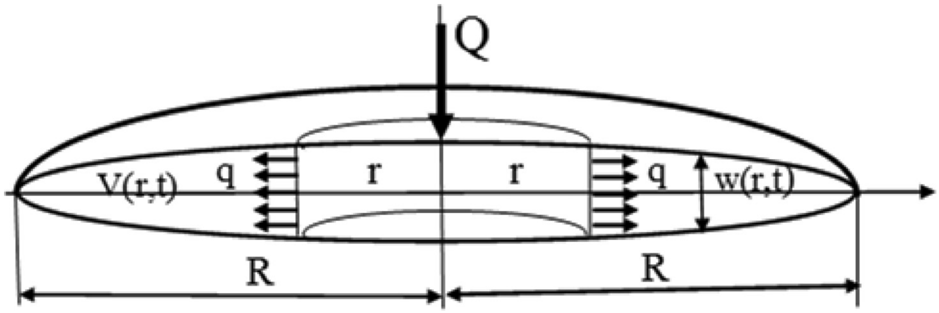

Fig. 1. A penny shape crack subjected to the pressure of fluid injected at the crack center.

based on time and space discretization of the original integro-differential equations were used. Note that the natural principal unknown of the problem is pressure distribution on the crack surface. By using time discretization procedure, the pressure distribution should be reconstructed at each step of the discrete process of crack growth. According to the author knowledge it was not mentioned in the literature that such a reconstruction is an ill-posed problem. Application of conventional numerical methods for solution of ill-posed problems can result substantial numerical errors, and only specific methods are efficient (see, e.g., Tikhonov & Arsenin, 1977). Because the ill-posed problem should be solved at each time step of the hydraulic fracture crack growth, the errors accumulate and a reliable solution can be lost. In addition to non-linearity and moving boundary, this is another principal difficulty in numerical solution of the problem. The fact that general hydraulic fracture problem is ill-posed, by its nature, was noted by Linkov (2011).

In the present work, the hydraulic fracture growth of a penny shape crack in homogeneous and isotropic elastic medium is considered. The fluid is injected in the crack center. Description of the crack growth is based on the lubrication equation (balance of the injected fluid and the crack volume), equation for crack opening in elastic media caused by fluid pressure distributed on the crack surface, the Poiseullie equation related local fluid flux with the crack opening and pressure gradient, and the classical criterion of crack propagation of linear fracture mechanics. Time discretization of these equations is interpreted as an actual process that consists of three stages: growth of the crack volume by a constant crack radius, an instant crack jump to a new radius, and filling the new appeared empty volume by the fluid. For solution of the ill-posed problem of reconstruction of the pressure distribution at each time step of crack growth, a specific class of approximating functions is used. These positive, monotonously decreasing functions are appropriate for approximation of actual pressure distributions and allow solving the ill-posed problem with sufficient accuracy. Comparison of the discrete model of hydraulic fracture crack growth with other solutions existing in the literature is presented. A simplified three-parametric model of hydraulic fracture crack growth is considered, and the region of application of this model is discussed.

2. A penny shape crack in a homogeneous and isotropic elastic medium subjected to fluid injection

We consider an infinite isotropic homogeneous elastic medium containing an isolated planar crack subjected to internal pressure p caused by fluid injected in the crack center with given injection rate $Q(t)$ (see Fig. 1). It follows from the symmetry of the problem that the crack remains planar and circular (penny shape) in the process of fluid injection. Crack opening $w(r, t)$ and pressure distribution $p(r, t)$ on the crack surface are functions of time t and the distance r from the crack center.

Let us introduce the fractional crack volume $v(r, t)$ by the equation

$$v(r,t) = 2\pi \int_r^{R(t)} w(x,t)x dx, \tag{1}$$

where $R(t)$ is the crack radius. Fractional crack volume $v(r, t)$ and crack opening $w(r, t)$ are related by the equation that follows from Eq. (1)

$$\frac{\partial v(r,t)}{\partial r} = -2\pi r w(r,t). \tag{2}$$

Let $q(r, t)$ be the fluid flux in the radial direction through the crack cross-section with coordinate r. For non-compressible fluid, the equation of balance of the fractional volume $v(r, t)$ and the injected fluid (lubrication equation) has the form

$$\frac{\partial v(r,t)}{\partial t} = 2\pi r q(r,t). \tag{3}$$

After differentiating this equation with respect to r and using Eq. (2) we obtain

$$\frac{\partial w(r,t)}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}[rq(r,t)]. \tag{4}$$

The fluid flux $q(r, t)$, crack opening $w(r, t)$, and pressure $p(r, t)$ relate by the Poiseuille law

$$q(r,t) = -\frac{w^3(r,t)}{\eta}\frac{\partial p(r,t)}{\partial r}. \tag{5}$$