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Oscillations of functionally graded microbeams

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ABSTRACT

The size-dependent oscillations of a third-order shear-deformable functionally graded microbeam are investigated taking into account all the longitudinal and transverse displacements and inertia as well as the rotation and rotary inertia. The modified couple stress theory along with the Mori–Tanaka homogenisation technique is employed to develop formulations for the elastic potential energy as well as the kinetic energy of the system. The energy of the system is balanced by the work of a harmonic excitation force via an energy method based on Hamilton's principle, yielding the size-dependent coupled nonlinear continuous models of the functionally graded system for the longitudinal and transverse displacements as well as the rotational motion. A model reduction procedure, on the basis of a weighted-residual method, is applied without any simplifications on the displacement/inertia/rotation. This operation yields three sets of second-order reduced-order coupled model of the functionally graded system for the longitudinal, transverse, and rotational motions. These reduced-order models are solved via use of a continuation method in order to construct the nonlinear frequency-response and force-response curves of the functionally graded system. A linear analysis is also performed by means of an eigenvalue extraction method in order to determine the linear natural frequencies of the system. It is shown that the material gradient index as well as the length-scale parameter of the functionally graded system affects the system dynamics substantially.

1. Introduction

Continuous microelements such as microbeams and microplates are the core elements of many microelectromechanical systems (MEMS) (Asghari, Kahrobaiyan, & Ahmadian, 2010; Baghani, 2012; Dehrouyeh-Semnani & Bahrami, 2016; Farokhi & Ghayesh, 2016a; Farokhi & Ghayesh, 2016b; Ghayesh & Farokhi, 2016a; Ghayesh, Farokhi, & Amabili, 2013a; Li and Hu, 2016; Rahaeifard, 2016). Functionally graded microscale continuous elements (Lü, Lim, & Chen, 2009) are a new class of electromechanical machine components which are gaining high interest mainly due to this fact that they are resistant to thermal and mechanical loadings at the same time; functionally graded continuous elements are a composite systems where ceramic and metal are combined using powder metallurgy technique – the ceramic is resistant to high temperature loads while the metal component is resistant to thermally induced fractures. There are many experimental investigations in the literature which show that the dynamical behaviour of continuous microelements is highly size-dependent (Akgöz & Civalek, 2013; Akgöz & Civalek, 2011; Dehrouyeh-Semnani, 2014; Dehrouyeh-Semnani, BehboodiJouybari, & Dehrouyeh, 2016; Ghayesh, Amabili, & Farokhi, 2013b; Hosseini & Bahaadini, 2016; Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012;

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Karparvarfard, Asghari, & Vatankhah, 2015; Kong, Zhou, Nie, & Wang, 2008; Mojahedi & Rahaeifard, 2016; Shafiei Kazemi, & Ghadiri , 2016a; Shafiei, Kazemi, & Ghadiri, 2016b; Taati, 2016); a phenomenon which cannot be predicted theoretically by the classical continuum mechanics – this paper employs the modified couple stress theory (Dai, Wang, & Wang, 2015; Dehrouyeh-Semnani, Dehrouyeh, Torabi-Kafshgari, & Nikkhah-Bahrami, 2015; Farokhi & Ghayesh, 2015a; Farokhi & Ghayesh, 2015b; Farokhi, Ghayesh, & Amabili, 2013a; Farokhi, Ghayesh, & Amabili, 2013b; Ghayesh & Amabili, 2014; Ghayesh & Farokhi, 2015a; Ghayesh, Farokhi, & Amabili, 2013c; Ghayesh, Farokhi, & Amabili, 2014; Gholipour, Farokhi, & Ghayesh, 2015; Li & Pan, 2015; Şimşek, 2010; Tang, Ni, Wang, Luo, & Wang, 2014) in order to take into account small-size influences (Farokhi, Ghayesh, Kosasih, & Akaber, 201a5) on the coupled dynamical response of the functionally graded system.

The literature on the dynamical/static analyses of functionally graded microbeams can mainly be divided into *two* main groups in terms of models being considered. In the *first* group, the system motion is analysed using either the *Euler-Bernoulli* or *Timoshenko* beam theories; however, in the *second* class, *higher-order* shear-deformation beam theories have been employed.

The literature on the *first* class is quite large. For example, Thai, Vo, Nguyen, and Lee (2015) employed the modified couple stress theory in order to examine the size-dependent buckling, bending, and free dynamics of functionally graded microbeams. Tajalli et al. (2013) employed a strain gradient elasticity theory in order to develop a formulation for a functionally graded Timoshenko microbeam. Nateghi and Salamat-talab (2013) analysed the effect of a thermal loading on the size-dependent buckling and free dynamics of a functionally graded microbeam via use of the modified couple stress theory. Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, (2012) employed the framework of a strain gradient elasticity in order to develop a size-dependent model of a functionally graded Euler–Bernoulli microbeam. Arbind and Reddy (2013) contributed to the field by developing nonlinear finite element models for the buckling analysis of both the Timoshenko and Euler–Bernoulli microscale beams via use of the modified couple stress theory. Rahaeifard, Kahrobaiyan, Ahmadian, and Firoozbakhsh (2013) developed a size-dependent model of an Euler–Bernoulli microbeam on the basis of a strain gradient elasticity theory; they examined the bending response as well as the free dynamics of the system. Ke, Wang, Yang, and Kitipornchai (2012) examined the free nonlinear dynamics of a functionally graded microbeam by means of the modified couple stress theory.

These studies were extended to *higher-order shear-deformations* (grouped in the *second* class); the literature on this group is *not* large. For example, Sahmani and Ansari (2013) employed a higher-order shear-deformation theory in order to obtain the size-dependent buckling behaviour of a functionally graded microbeam in the presence of a thermal loading by means of the generalised differential quadrature method. On the basis of the modified couple stress theory, Şimşek & Reddy (2013) developed a mathematical model for the buckling analysis of an elastically constrained functionally graded microbeam via use of a unified higher-order beam theory. Zhang, He, Liu, Gan, and Shen (2014) examined the size-dependent buckling response, bending, and free dynamical behaviour of a functionally graded microbeam on the basis of a strain gradient elasticity theory together with an improved version of a third-order shear-deformation theory. Sahmani, Bahrami, and Ansari (2014) employed a modified version of the strain gradient elasticity theory in order to examine the nonlinear free dynamics of the system. Ansari, Shojaei, and Gholami (2016) analysed the nonlinear size-dependent dynamical behaviour of a third-order shear-deformable functionally graded microbeam by means of the variational differential quadrature method and a strain gradient elasticity theory.

Contributions of this paper to the field: This paper, for the first time, analyses the nonlinear size-dependent coupled rotational-transverse-longitudinal motion characteristics of a third-order shear-deformable functionally graded microbeam based on the modified couple stress theory by means of a continuation method by providing a stability analysis together with bifurcation types for a high-dimensional system. More specifically, the works of damping and external excitation, the size-dependent potential energy, and the kinetic energy of the system are developed in terms of the system parameters and the displacement field. An energy balance on the basis of Hamilton's principle is employed in order to obtain the coupled rotational-transverse-longitudinal continuous model of the system. Based on a weighted-residual method, the model is reduced and then solved by means of the pseudo-arclength continuation method; the Floquet theory is employed for the stability analysis of the system in motion. The size-dependent frequency-response and force-response curves of this functionally graded system are constructed for all the rotational, transverse, and longitudinal motions.

2. Model development and solution method

The schematic of a two-phase (ceramic-metal) functionally graded shear-deformable microbeam is depicted in Fig. 1. The geometry of the microbeam is shown by the following parameters: length L, thickness h, width b, and cross-sectional area A which is assumed to be uniform along its length. The microbeam is considered to be hinged at both ends and subjected to a transverse harmonic load per unit length of $F(x)\cos(\omega t)$ (in the z direction), where t is time and ω denotes the excitation frequency. The Mori-Tanaka homogenisation scheme is used in order to obtain the effective material properties on the basis of the ceramic and metal properties; the Mori-Tanaka scheme is almost the most applicable method in describing the local effective properties of functionally graded materials (Şimşek & Reddy, 2013). The mixture of ceramic and metal changes continuously from ceramic at the bottom surface ($z=h/2$) to metal at the top surface ($z=-h/2$). Using the powder metallurgy, different distributions can be obtained along the microbeam thickness (i.e. the z direction). As a result, different material properties of the mixture such as mass density ρ, Young's modulus E, Poisson's ratio υ, and shear modulus μ vary continuously along the z coordinate.