International Journal of Engineering Science 115 (2017) 43–50

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Time domain scattering of elastic waves by a cavity, represented by radiation from equivalent body forces

Chao Yang[a], Jan D. Achenbach[b,*]

[a] School of Mechanical Engineering, Beijing Institute of Technology, Beijing 100081, China
[b] McCormick School of Engineering and Applied Science, Northwestern University, Evanston, IL 60208, United States

ARTICLE INFO

Article history:
Received 11 January 2017
Accepted 10 March 2017

Keywords:
Scattering
Cavity
Equivalent body forces

ABSTRACT

A method of representing scattering in the time domain by a cavity in an elastic body of infinite extent by radiation from equivalent body forces is presented. It is shown that the equivalent body forces are a combination of double and single forces. The equivalent forces are determined in terms of properties of the incident wave, the solid's elastic constants and the volume of the scatterer, which may be of arbitrary shape. Once the equivalent forces have been determined, the scattered field is determined by the use of a Green's function. Scattering by cavities of 2D and 3D general shape is considered and analyzed. Two special cases, namely, a circular cavity and a spherical cavity, are used to illustrate the method. The results are valid for large measurement distances and waves that are long relative to a characteristic length of the cavity. The method can easily be extended to scattering by cavities in waveguides.

1. Introduction

Scattering occurs when an elastic wave impinges on a cavity in a solid body. Scattering by a cavity is of great interest, and has been widely studied for quantitative nondestructive evaluation, since cavities are common defects in materials and components. By studying the scattering from a cavity, information on the size, shape and orientation of the cavity may be obtained.

Classically, the problem of the scattering by a cavity in a homogeneous, linearly elastic body can be analyzed by employing the scalar and vector potentials for the displacement fields (Achenbach, 1973). Three methods of analysis for this problem, including the method of wave functions expansion, the integral equation method and the integral transform method were discussed in some detail by Mow and Pao (1971). A matrix method was developed by Waterman (1969), and extended by Varatharajulu and Pao (1976) and Visscher (1980). Coussy (1984), used a perturbation method to treat the scattering problem. For more references on the problem of scattering by a cavity in an elastic solid, we refer to Pao and Mow (1963), Ying and Truell (1956) and Jain and Kanwal (1980) for incident plane longitudinal waves; Einspruch, Witterholt, and Truell (1960), Kraft and Feanzblau (1971) and Norris (1986) for plane transverse waves; and Grahn (2003), Karlsson (1984), Moreau, Caleap, Velichko, and Wilcox (2012) and Moreau, Caleap, Velichko, and Wilcox (2011) for lamb waves.

In this paper we consider the scattering in the time domain of an incident pulse by a cavity of general shape in a homogenous, isotropic, linearly elastic body. Generally, the analytical analysis of the scattering problem is complicated, es-

* Corresponding author.
E-mail addresses: yangchaobit@gmail.com (C. Yang), achenbach@northwestern.edu (J.D. Achenbach).

http://dx.doi.org/10.1016/j.ijengsci.2017.03.005
0020-7225/

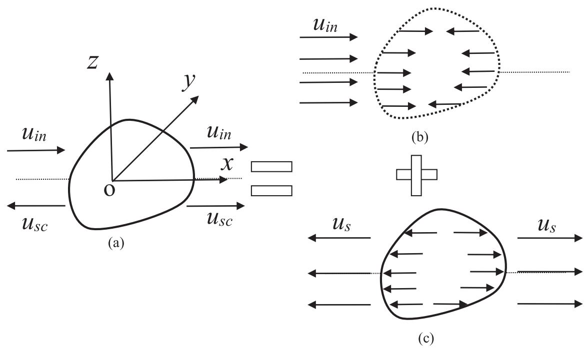

Fig. 1. The superposition principle for scattering from a cavity.

pecially for scatterers of general shape, while numerical treatments, such as the finite element method, the discreet element method and the boundary element method, may encounter serious instability problems and may require an extensive numerical effort. An analytical analysis becomes feasible if some approximations are introduced. For long waves, three approximations used to investigate the scattering problem are the Born approximation (Gubernatis, Domany, Krumhansl, & Huberman, 1977; Huang, Schmerr, & Sedov, 2006), the quasistatic approximation and the extended quasistatic approximation (Gubernatis, 1979; Mal & Knopoff, 1968). The Born approximation, which was adopted from quantum mechanics, uses the incident fields to replace the fields inside the cavity, and makes it possible to treat cavities of general shape. Its utility is, however, restricted to backward scattered directions in the case of cavities. The quasistatic and the extended quasistatic approximations use a static strain field to replace the actual strain field inside the cavity, but it is only easily applicable to simple shapes due to the complexity and difficulties of the analysis.

To simplify the analysis, for wavelengths sufficiently larger than the largest characteristic length dimension of the cavity, we represent the scattering by a cavity by radiation from equivalent body forces in the undamaged body. These equivalent body forces produce radiation equal to the scattered field. This method may not provide information on the shape and orientation of the cavity, but it can provide information on the volume of the cavity, which is still very useful to quantitative nondestructive evaluation and some other applications. Additionally, this representation is quite straightforward and the scattered field can be expressed in a simple form, using a Green's function, and can easily be extended to waveguides.

In Sections 2 and 3 the forms of equivalent body forces, which are both double and single forces, are derived for a cavity of 2D general shape, and illustrated for the special case of a circular cavity. The equivalent body forces for a cavity of 3D general shape are derived in Section 4, and an example for the spherical cavity is also presented. The scattered field is then obtained in the time domain in Section 5 by the use of a Green's function.

2. Formulation of the scattering problem

Fig. 1(a) shows the configuration of a wave, u_{in}, incident on a cavity. The incident wave generates a field of scattered waves, u_{sc}, propagating in all directions. The total field can be written as

$$u_{tot}(\mathbf{x}) = u_{in}(\mathbf{x}) + u_{sc}(\mathbf{x}). \tag{1}$$

For purposes of explanation, we consider the case of a longitudinal wave that is being scattered by a 2D cavity. By virtue of linear superposition, the scattered field is equivalent to the field generated by the application of tractions on the surface of the cavity. These tractions are equal in magnitude but opposite in sign to corresponding tractions that are obtained from the stress components generated by the incident wave on the boundary of a "virtual" cavity of the same shape as the actual cavity in the undamaged body. The virtual cavity is shown in Fig. 1(b), and (c) shows the corresponding loading of the actual cavity. Superposition of the results of Fig. 1(b) and (c) shows that scattering is defined as the solution for the cavity loading shown in Fig. 1(c).

We consider an incident wave of the form

$$u_x = Uf(t - x/c_L) = \frac{U}{\pi}\mathrm{Re}\int_0^\infty \hat{f}(\omega)e^{-i\omega(t-x/c_L)}d\omega, \tag{2}$$

where $f(t)$ is a dimensionless function, and c_L is the velocity of longitudinal waves. The function $f(t - x/c_L)$ has been conveniently decomposed as an integration of harmonics, where

$$\hat{f}(\omega) = \int_0^\infty f(t)e^{i\omega t}dt. \tag{3}$$