International Journal of Engineering Science 115 (2017) 117–130

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

# Dynamics of functionally graded micro-cantilevers

Hamed Farokhi[a], Mergen H. Ghayesh[b,*], Alireza Gholipour[b]

[a] Department of Mechanical Engineering, McGill University, Montreal, Quebec H3A 0C3, Canada
[b] School of Mechanical Engineering, University of Adelaide, South Australia 5005, Australia

ARTICLE INFO

*Article history:*
Received 18 January 2017
Accepted 21 January 2017

*Keywords:*
Functionally graded micro-cantilever
MCST
Dynamics
MEMS

ABSTRACT

The nonlinear size-dependent dynamics of a functionally graded micro-cantilever is investigated when subject to a base excitation resulting in large-amplitude oscillations. A geometric nonlinearities due to large changes in the curvature is taken into account. Employing the Mori–Tanaka homogenisation technique (for the material properties), the modified couple stress theory (MCST) is used to formulate the potential and kinetic energies of the system in terms of the transverse and axial motions. A dynamic energy balance is performed between the energy terms, yielding the continuous models for the axial and transverse displacements. The inextensibility condition results in the size-dependent model of the functionally graded micro-cantilever involving inertial and stiffness nonlinear terms. The resultant model is discretised based on a weighted-residual technique yielding a high-dimensional truncated model (required for accurate simulations). A parameter-continuation scheme together with a time integration method is introduced to the truncated model so as to determine the resonances with stable and unstable solution branches with special consideration to the effect of different system parameters, such as material gradient index and the length-scale effect on the nonlinear dynamics of the functionally graded micro-cantilever.



## 1. Introduction

Cantilevers of microscale dimensions form the core components of many microelectromechanical systems (MEMS) (Asghari, Kahrobaiyan, & Ahmadian, 2010; Baghani, 2012; Ghayesh & Farokhi, 2015a, b; Ghayesh, Farokhi, & Amabili, 2013a, b; Zhang & Meng, 2007; Zheng, Dong, Lee, & Gao, 2009). Experiments showed that their deformation behaviour is highly size-dependent (Akgöz & Civalek, 2013a, b, Akgöz & Civalek, 2011; Dehrouyeh-Semnani, 2014; Dehrouyeh-Semnani, Behbood-iJouybari, & Dehrouyeh, 2016; Farokhi, Ghayesh, Gholipour, & Hussain, 2017; Ghayesh, Amabili, & Farokhi, 2013a, b, Hosseini & Bahaadini, 2016; Kahrobaiyan, Rahaeifard, Tajalli, & Ahmadian, 2012; Karparvarfard, Asghari, & Vatankhah, 2015; Kong, Zhou, Nie, & Wang, 2008; Mojahedi & Rahaeifard, 2016; Shafiei, Kazemi, & Ghadiri, 2016a, b, Taati, 2016). Theoretically, only the advanced continuous theories such as strain gradient and MCST (Dai, Wang, & Wang, 2015; Dehrouyeh-Semnani et al., 2015; Farokhi & Ghayesh, 2015 b, Farokhi & Ghayesh, 2015 a, Farokhi, Ghayesh, & Amabili, 2013a, b, Ghayesh & Amabili, 2014; Ghayesh and Farokhi, 2015, Ghayesh, et al., 2013 a, b, Ghayesh, Farokhi, & Amabili, 2014; Ghayesh, Farokhi, & Hussain, 2016; Gholipour, Farokhi, & Ghayesh, 2015; Li & Pan, 2015; Şimşek, 2010; Tang, Ni, Wang, Luo, & Wang, 2014) theories are capable of incorporating size effects in the dynamical modelling of microscale elements; in other words, simulations based on the classical continuum mechanics may result in drastic errors (Farokhi, Ghayesh, & Amabili, 2013).

* Corresponding author.
*E-mail address:* mergen.ghayesh@adelaide.edu.au (M.H. Ghayesh).

http://dx.doi.org/10.1016/j.ijengsci.2017.01.004
0020-7225/

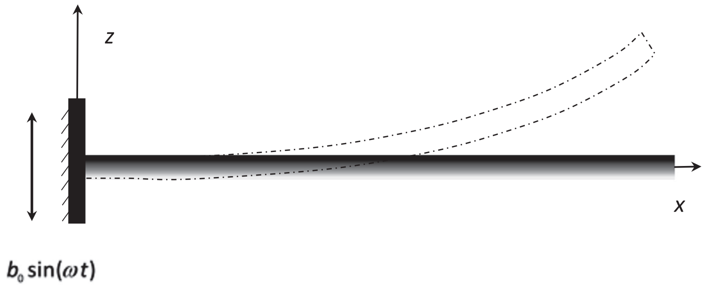


**Fig. 1.** Schematic representation of functionally graded micro-cantilever.

In recent years, microstructures made of functionally graded (FG) materials have gained high interests in MEMS industry due to their mechanical properties to achieve a better performance and sensitivity (Witvrouw, 2005; Huang, 2008; Lü, Lim, & Chen, 2009).

*Micro-cantilevers*, as opposed to *supported–supported* (i.e. combination of clamps and pins) microbeams (Farokhi & Ghayesh, 2016), are subjected to large deformations when a transverse load is applied; this is due to the fact that one end of the microsystem possesses displacement-freedom. In other words, the effect of the curvature-related nonlinearities become dominant and should not be neglected, especially when coupled with the length-scale parameter.

The literature on the dynamics/statics of *supported-supported functionally graded* microbeams, involving both linear and nonlinear analyses is large; for example, Akgöz and Civalek (2014) analysed the thermal variation influences on the buckling behaviour of a FG microbeam resting on an elastic foundation. Tajalli et al. (2013) formulated the motion behaviour of a FG Timoshenko microbeam. Aghazadeh, Cigeroglu, and Dag (2014) contributed to the field by developing a procedure of obtaining the linear dynamic/static characteristics of Timoshenko, Bernoulli–Euler, and shear deformable types of FG microbeams. Arbind and Reddy (2013) developed a geometrically nonlinear finite element model for a FG microbeam employing the Euler–Bernoulli and Timoshenko theories via use of MCST; the resultant model was used to obtain the buckling response of the microsystem.

The literature on the dynamics/statics of *functionally graded micro-cantilevers* is not large. Asghari, Ahmadian, Kahrobaiyan, & Rahaeifard (2010) examined the size-dependent static and free dynamics of FG micro-cantilevers based on MCST. Shafiei et al. (2016a, b) examined the transverse vibrations of an axially FG micro-cantilever based on the Euler–Bernoulli and MCST. Akgöz & Civalek (2013a, b) employed MCST in order to obtain the dynamical response of non-uniform axially functionally graded micro-cantilevers.

This paper is the first which examines the size-dependent nonlinear dynamics of a *functionally graded micro-cantilever undergoing large oscillations* via MCST incorporating curvature and inertial nonlinearities. The size-dependent potential energy of the functionally graded micro-cantilever is obtained as a continuous function of the system parameters employing the Mori–Tanaka technique as well as constitutive relations. A dynamic balance is introduced to the kinetic energy and the size-dependent potential energy giving the continuous equations of the transverse and axial displacements. The inextensibility condition leads to an integro-partial-differential equation of motion of the FG micro-cantilever. A system reduction is carried out on the basis of the Galerkin scheme. The resonances of the functionally graded micro-cantilever are obtained highlighting the influence of different microsystem parameters including the length-scale parameter as well as the material gradient index.

## 2. Microsystem model of functionally graded micro-cantilever

Consider a functionally graded micro-cantilever of length $L$, thickness $h$, and cross-sectional area of $A$ (Fig. 1). The left end of the functionally graded micro-cantilever is clamped while the right end is free to move. The clamped end is excited harmonically by $b_0\sin(\omega t)$ where $\omega$ is the excitation frequency, $t$ is time, and $b_0$ denotes the excitation amplitude. The total displacement field is shown by the axial component $u(x,t)$ and the transverse component $w(x,t)$, where $x$ is axial coordinate. The functionally graded micro-cantilever is made of metal and ceramic and the mechanical properties are Young's modulus $E$, mass density $\rho$, Poisson's ratio $\upsilon$, and shear modulus $\mu$. At $z=h/2$ (i.e. at the top surface) the micro-cantilever is ceramic-rich and at $z=-h/2$ (the bottom surface) is metal-rich.

The Mori–Tanaka homogenisation technique is utilised to determine the effective mechanical properties of the mixture. As such, the effective bulk modulus (shown by $K_e$)and the effective shear modulus (shown by $\mu_e$) are defined as functions of $\mu_c$, $\mu_m$, $K_c$, and $K_m$ (shear modulus of the ceramic, shear modulus of the metal, bulk modulus of the ceramic, and the bulk modulus of the metal, respectively) as well as $v_m$ and $v_c$ (the volume fractions of the metal and ceramic, respectively where $v_m + v_c = 1$); thus

$$\frac{K_e - K_m}{K_c - K_m} = \frac{\upsilon_c}{1 + \upsilon_m (K_c - K_m)/(K_m + 4\mu_m/3)} , \tag{1}$$