International Journal of Engineering Science 114 (2017) 16–40

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Micromechanics of dislocations in solids: ***J***-, *M*-, and ***L***-integrals and their fundamental relations

Eleni Agiasofitou, Markus Lazar*

Heisenberg Research Group, Department of Physics, Darmstadt University of Technology, Hochschulstr. 6, D-64289 Darmstadt, Germany

A R T I C L E   I N F O

Article history:
Received 31 August 2016

Keywords:
Dislocations
Peach–Koehler force
Rotational moment
Configurational work
Interaction energy
Energy-release

A B S T R A C T

The aim of the present work is the unification of incompatible elasticity theory of dislocations and Eshelbian mechanics leading naturally to Eshelbian dislocation mechanics. In such a unified framework, we explore the utility of the ***J***-, *M*-, and ***L***-integrals. We give the physical interpretation of the *M*-, and ***L***-integrals for dislocations, connecting them with established quantities in dislocation theory such as the interaction energy and the ***J***-integral of dislocations, which is equivalent to the well-known Peach–Koehler force. The ***J***-, *M*-, and ***L***-integrals for dislocations have been studied in the framework of three-dimensional, incompatible, linear elasticity. First of all, the general formulas of the ***J***-, *M*-, and ***L***-integrals for dislocations are given. Next, the examined integrals are specified for straight dislocations. Finally, the explicit formulas of the ***J***-, *M*-, and ***L***-integrals are calculated for straight (screw and edge) dislocations in isotropic materials. The obtained results reveal the physical interpretation and significance of the *M*-, and ***L***-integrals for straight dislocations. The *M*-integral between two straight dislocations (per unit dislocation length) is equal to the half of the interaction energy of the two dislocations (per unit dislocation length) depending on the distance and on the angle, plus twice the corresponding pre-logarithmic energy factor. The L_3-integral between two straight dislocations is the *z*-component of the configurational vector moment or the rotational moment about the *z*-axis caused by the interaction between the two dislocations. Fundamental relations between the ***J***-, *M*-, and L_3-integrals are derived showing the inherent connection between them. The relations connecting directly the ***J***-, *M*-, and L_3-integrals with the interaction energy are obtained. These relations have been proven to be of great significance. Since based on them; the interpretation of the ***J***-, and L_3-integrals as translational and rotational energy-release, respectively, is achieved, and secondly a stability criterion for straight edge dislocations is formulated in terms of the J_k-integral, revealing the physical importance of the considered integrals.

1. Introduction

Dislocations are one of the most important defects in solids, since they influence the properties of a crystal; not only the mechanical, but also the electric, magnetic, optic, and semi-conducting properties as well as the growth of a crystal (e.g., Nabarro, 1967). Dislocations cause plasticity and hardening in crystals (e.g., Cottrell, 1953 and Lardner, 1974). Consequently, the physics of the interaction between dislocations is an important research field and for that reason this work

* Corresponding author.
E-mail addresses: agiasofitou@mechanik.tu-darmstadt.de (E. Agiasofitou), lazar@fkp.tu-darmstadt.de (M. Lazar).

http://dx.doi.org/10.1016/j.ijengsci.2017.02.001

is focused on the study of the ***J***-, *M*-, and ***L***-integrals between two dislocations. The interaction force between dislocations is the so-called Peach–Koehler force (Peach & Koehler, 1950). Rogula (1977) and Kirchner (1999, 1984) gave a field-theoretical derivation of the Peach–Koehler force as the divergence of the so-called Eshelby stress tensor (Eshelby, 1975), using the framework of dislocation theory including incompatible distortion tensors. Rice (1985),Weertman (1996) and Kirchner (1999) found that the Peach–Koehler force is equivalent to the ***J***-integral of dislocations. In general, the Peach–Koehler force is a configurational or material force. Material forces are forces acting on fields in the sense of Lorentzian forces, for example the static Lorentz force is the interaction force on an electric charge in the presence of an electric field. Material forces should not be confused with forces in the sense of Newtonian forces, acting on masses in the second Newton law; where a force is defined as mass times acceleration (see also Kröner, 1993 and Maugin, 1993). In elasticity, Newtonian forces appear as the divergence of the Cauchy stress tensor, whereas material forces appear as the divergence of the Eshelby stress tensor (static energy-momentum tensor). Material forces are forces that do act on defects such as dislocations, disclinations, point defects, cracks, etc. The physical meaning of the Peach–Koehler force as the interaction force acting on a dislocation, defined by the dislocation density tensor due to the stress field caused by another dislocation (or another defect) in an elastic material, is well-established in classical dislocation theory, and can be found in many textbooks on dislocations (e.g., Landau and Lifschitz, 1986,Lardner, 1974,Hirth and Lothe, 1982 and Li & Wang, 2008). In the framework of the dislocation gauge field theory, Agiasofitou and Lazar (2010) proved that the Peach–Koehler force is, in particular, the interaction force between the elastic and the dislocation subsystems enlightening that the Peach–Koehler force is self-equilibrating. Moreover, they have proven that the Peach–Koehler force is a material force caused by dislocations as incompatibilities.

The ***J***-integral became originally famous in fracture mechanics. Rice (1968) used the Eshelby stress tensor to derive a certain conservation law which is known as the *J*-integral. Seven conservation laws of elasticity, which are related to translational, rotational, and scaling symmetries, were originally derived by Günther (1962), and Knowles and Sternberg (1972). The corresponding conservation integrals are the ***J***-, ***L***-, and *M*-integrals introduced by Budiansky and Rice (1973). Budiansky and Rice (1973) were the first to give a physical interpretation to the ***J***-, ***L***-, and *M*-integrals as the energy-release rates per unit cavity translation, rotation, and expansion, respectively. An important contribution to connect the ***J***-, ***L***-, and *M*-integrals with the energy-release rates for cracks and the relations between them, was given in a series of papers by Golebiewska-Herrmann and Herrmann (1981b),Pak, Golebiewska-Herrmann, and Herrmann (1983), and Eischen and Herrmann (1987). Golebiewska-Herrmann and Herrmann (1981b) showed that the *L*-integral represents the rotational energy-release (rate)[1] induced from the rotation of a central plane crack. Chen (2002) (see also Chen & Lu, 2003) was the first to show that the *M*-integral equals twice the change of the total potential energy owed to single cracking of a central crack in a plane elastic body. This conclusion was implied by Budiansky and Rice (1973) and Golebiewska-Herrmann and Herrmann (1981b), but written explicitly by Chen (2002) emphasizing for the first time this physical interpretation of the *M*-integral. An interesting overview about conservation laws, ***J***-, ***L***-, and *M*-integrals as well as related concepts with various applications to defect mechanics in the framework of compatible linear elasticity is given by Kienzler and Herrmann (2000) and Chen (2002).

Using elasticity as field theory, Lazar and Kirchner (2007a) derived the ***J***-, ***L***-, and *M*-integrals for gradient elasticity theory of non-homogeneous, incompatible, linear, anisotropic media. For the first time deriving the general expressions of the ***L***-, and *M*-integrals for dislocations, for both elasticity theory and gradient elasticity theory. For compatible micropolar elasticity, the ***J***-, ***L***-, and *M*-integrals were given by Lubarda and Markenscoff (2003). For incompatible micropolar elasticity including dislocations and disclinations, the ***J***-, ***L***-, and *M*-integrals were derived by Lazar and Kirchner (2007b). For incompatible micromorphic elasticity with dislocations, disclinations, and point defects the ***J***-integral including the Peach–Koehler force as well as the Mathisson–Papapetrou force were given by Lazar and Maugin (2007). As mentioned above, the ***J***-integral of dislocations in incompatible elasticity is equivalent to the Peach–Koehler force, which appears as the divergence of the Eshelby stress tensor. In the framework of configurational or Eshelbian mechanics, Lazar and Kirchner (2007a) showed that for dislocations the ***L***-integral is equivalent to configurational or material vector moment, which appears as the divergence of the angular momentum tensor and the *M*-integral is equivalent to configurational or material work, which appears as the divergence of the scaling flux vector of dislocations.

The question that arises now is what is the specific physical meaning of the *M*-, and ***L***-integrals for dislocations. The first single work posing the question "What is *M* for a dislocation?" goes back to Rice in 1985. Rice (1985) studied the *M*-integral when centered on a dislocation line in the framework of two-dimensional, compatible, linear elasticity and found that the *M*-integral is equal to the "dislocation energy factor" (see also Asaro & Lubarda, 2006). However, a dislocation is a line defect in a three-dimensional crystal and the expression of the *M*-integral is different in two- and three-dimensions, since the *M*-integral depends on the space dimension. In this work, we calculate explicitly the *M*-integral of two dislocations in the framework of three-dimensional, incompatible, linear elasticity revealing its real physical meaning. We show for the first time in dislocation theory that the *M*-integral between two straight dislocations (per unit dislocation length) is equal to the half of the interaction energy of the two dislocations (per unit dislocation length), which depends on the distance and on the angle, plus twice the corresponding pre-logarithmic energy factor.[2] Therefore, the *M*-integral of two straight dislocations

[1] Attention should be payed to the terminology "energy-release rate", since in the literature the expression "rate" is often used abusively in statics. For reasons of clarity, the terminology "energy-release" is used in this work, since it is in the framework of elastostatics; letting the terminology "energy-release rate" available for elastodynamics.

[2] In this work, we use the standard definition of the pre-logarithmic energy factor given in the literature of dislocation theory (see, e.g., Hirth and Lothe, 1982 and Teodosiu, 1982) and it corresponds to the "dislocation energy factor" used in Rice (1985).