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Size dependent dynamic behavior of electrostatically actuated microbridges

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ABSTRACT

In this paper, based on the size dependent theories, an accurate investigation for nonlinear dynamic analysis of electrostatically actuated microbridges is presented by using the nonlinear finite element formulation, analytical method and numerical approach. The microbridge used as a microresonator, is driven by simultaneously applied DC and AC voltages. In the framework of modified couple stress theory (MCST), the nonlinear equation of dynamic motion for this system is derived using extended Hamilton's principle. Thereupon, the nonlinear partial differential equation is converted to the nonlinear ordinary equation by Galerkin's method and solved by analytical and numerical techniques. The analytical solution for nonlinear vibration of the microresonators is obtained by perturbation theory. Findings show that the numerical and analytical results are in good agreement with each other. In order to make further verification of the analytical expression, a nonlinear nonclassical finite element formulation for dynamic deformation of the system is presented to calculate the time history results. The good agreement observed between the results of analytical method and nonlinear finite element simulation demonstrates that the procedures used in the analytical method such as Taylor expansion, one mode analysis and perturbation expansion in multiple scale approach are appropriate. By determining the frequency-response curves, the effects of size dependency on the amplitude of vibration and frequency position of the peak response are studied. Results show that considering the size dependent theory leads to notable decreasing of the amplitude of nonlinear vibration and approaching the frequency value of peak response to linear resonance frequency.

1. Introduction

Microresonators are miniaturized electromechanical systems which can be utilized for mass detecting, biological mass sensing such as viruses or bacterias sensing, microgyroscopes, signal filtering and mass inertial sensors. They are operated at high frequencies that may actuate the resonance vibrations of the microresonator structures (Hassanpour, Esmailzadeh, Cleghorn, & Mills, 2010; Miandoab, Pishkenari, Yousefi-Koma, & Tajaddodianfar, 2014; Radzio, Oesterschulze, & Korsch, 2013). Microresonators also can be used as high sensitivity mass sensors that have variable natural frequency depending on mass variations due to accumulation or detachment of the particles. In the MEMS resonators, high sensitivity can be achieved by causing the resonator structures to be miniaturized. In addition to this, having low power consumption, low manufacturing expenditure due to batch production, small size and compatibility with integrated circuit fabrication process can lead to widespread application of the microresonators in high-precision mass sensors, motion sensing and signal filtering (Ibrahim, & Younis, 2009; Rahafrooz, & Pourkamali, 2010). Among various operating mechanisms, electrostatic and piezoelectric ac-

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tuations are mostly utilized in the MEMS resonator transducers. The piezoelectric resonator requires the attachment of the piezoelectric film to micromechanical components, hence the quality factor of this resonator is decreased notably in this situation (Rahafrooz, & Pourkamali, 2010). The electrostatic mechanism requires thin gaps with complicating fabrications in order to increase the attractive forces. However, due to high efficiency, fast response, simplicity and integrability with various circuits, it is of interest to implement this mechanism in the micromachined resonators (Nayfeh, & Younis, 2005; Rahafrooz, & Pourkamali, 2010). The building block of electrostatically actuated MEMS resonators are mainly composed of microbridges which are deflected by the bias DC voltage and vibrated by harmonic exciting force due to AC component of voltage. Since the exciting frequency of AC voltage is nearly identical to first bending natural frequency of the microbridge, the microresonators are vibrated under primary resonance condition. Due to the importance of the oscillatory and instability analysis of the microresonators, many researches have been done on the dynamic, vibration and instability of MEMS resonators and bridges (Alsaleem, Younis, & Ouakad, 2009; Alsaleem, Younis, & Ruzziconi, 2010; Guo, Wang, & Rogerson, 2012; Hassanpour, Esmailzadeh, Cleghorn, & Mills, 2010; Ibrahim, & Younis, 2009; Miandoab et al., 2014; Mojahedi, Ahmadian, & Firoozbakhsh, 2013; Mojahedi, Ahmadian, & Firoozbakhsh, 2014; Mojahedi, Zand, Ahmadian, & Babaei, 2011; Nayfeh, & Younis, 2005; Radzio, Oesterschulze, & Korsch, 2013; Rahafrooz, & Pourkamali, 2010). Abdel-Rahman, Younis, and Nayfeh (2002) presented a nonlinear model for pull-in instability and vibration behavior of microbridges actuated by electrostatic mechanism. They investigated the effect of nonlinear mid-plane stretching on the natural frequencies and pull-in instability voltage using abovementioned model. Nayfeh, and Younis (2005) investigated the nonlinear dynamics of electrostatically actuated microbeam resonators under subharmonic and superharmonic resonance excitations. They showed that the frequency-response of the microbeam will be subjected to pull-in instability, when the subharmonic resonance is found in the nonlinear analysis of the microresonators. By considering thermoelastic damping, the out of plane vibration of circular plate resonators was developed by Sun, and Saka (2010). In their research, the analytical expression for thermoelastic damping has been presented and the effects of plate dimension, ambient temperature and boundary conditions on the thermoelastic damping have been investigated. Ibrahim, and Younis (2009) studied experimentally and theoretically the influence of mechanical shock on the dynamic response of electrostatically actuated parallel-plate resonators. The flexural vibrations of beam resonators caused by thermal and mechanical loads were investigated by Sharma, and Kaur (2015). In their modeling, the Euler-Bernoulli theory for the beam and non-Fourier heat transfer model for heat conduction have been considered. Azizi, Chorsi, and Bakhtiari-Nejad (2016) studied the subharmonic resonance (order of one-half) of electrostatically actuated microbeam resonators sandwiched by piezoelectric layers.

Experimental findings show that the classical theory cannot predict the mechanical behavior of micorscale structures (Fleck, Muller, Ashby, & Hutchinson, 1994; Lam, Yang, Chong, Wang, & Tong, 2003; McFarland, & Colton, 2005; Stölken, & Evans, 1998). The stiffness of the micro-scale structures predicted by classical continuum theory is considerably smaller than the actual value of microstructure stiffness measured in experiment. The researcher's observations show that the nondimesionalized static and dynamic deformations of the micro-scale systems which are independent of the structure size in classical theory, are considerably changed by varying the dimensions of the structure. Also the difference between the results of classical theory and experiments increases as the size of structure decreases (Fleck et al., 1994; Lam et al., 2003; McFarland, & Colton, 2005; Stölken, & Evans, 1998). In the past decade, the size dependent theories have been proposed to accurately predict the mechanical behavior of the micro scale structures (Shaat, & Mohamed, 2014; Yang, Chong, Lam, & Tong, 2002). The most famous theories related to size dependent effects are couple stress and strain gradient theories (Lam et al., 2003; Yang et al., 2002). The couple stress theory based on the linear elasticity was devised by Mindlin, and Tiersten (1962) as a nonclassical continuum theory. This theory has two material length scale parameters which must be determined by experiment. Finding these parameters and solving the complex equations cause difficulty in using the theory. In order to reduce these difficulties, modified couple stress theory (i.e. a special case of the couple stress theory) was proposed by Yang et al. (2002). The presented modified couple stress theory (MCST) satisfies the moment of couple's equations. In this theory, in addition to the stresses caused by forces, the couple stresses caused by moments are considered in the calculation of the potential energy (Akgöz, & Civalek, 2011; Akgöz, & Civalek, 2014; Asghari, 2012; Asghari, Kahrobaiyan, & Ahmadian, 2010). The symmetry of the couple stress tensor, having one material length scale parameter and accurately predicting of the experimental results are advantages of utilizing the modified couple stress theory in simulation of the mechanical behavior of the MEMS structures (Dehrouyeh-Semnani, Dehrouyeh, Zafari-Koloukhi, & Ghamami, 2015; Ghayesh, Farokhi, & Alici, 2016; Guo, Chen, & Pan, 2016; Hosseini, & Bahaadini, 2016; Shafiei, Mousavi, & Ghadiri, 2016). Effect of size dependency on the Euler- Bernoulli microbeams under static loading was assessed by Park, and Gao (2006). They used the modified couple stress theory to study the static deflection of the microbeam and showed that the bending deformations of the beam predicted by this theory is smaller than those predicted by classical theory. The analytical expressions for static deflection and natural frequencies of the FGM microbeams were presented by Asghari, Ahmadian, Kahrobaiyan, and Rahaeifard (2010), based on the MCST. On the basis of modified couple stress theory, Rahaeifard, Kahrobaiyan, Ahmadian, and Firoozbakhsh (2012) and Rahaeifard, Ahmadian, and Firoozbakhsh (2013, 2015) studied the static, vibration and pull-in instability of the electrostatically driven microbeams. Shaat, and Mohamed (2014) studied the effects of surface energy on the static behavior of electrostatically actuated microbeams considering modified couple stress theory. They have used the Gurtin and Murdoch model to assess the surface effects. Mojahedi, and Rahaeifard (2016) presented a nonlinear formulation for coupled 3D dimensional motions of the microbeams considering MCST. With regard to the presented model, they studied the static and free vibration of the 3D microbeam and showed that applying load in one bending direction leads to deformation variations on other bending direction. Li, Li, and Hu (2016) developed a new model for Timoshenko beam consisting of FGM based on