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Elastoplastic constitutive equation of metals under cyclic loading

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ABSTRACT

The subloading surface model is based on the natural concept that the plastic strain rate develops continuously as the stress approaches the yield surface, in which an elastic domain is not assumed in this model. It therefore always describes the continuous variation of the tangent modulus. It does not require the determination of the offset value for yielding and the incorporation of an algorithm for the judgment of yielding, i.e. judgment of whether or not the stress reaches the yield surface. In addition, the stress is always attracted to the yield surface in the plastic loading process and thus it is automatically pulled-back to the yield surface when it goes out from the yield surface. In this article, complete elastoplastic constitutive equation of metals is formulated within the framework of the subloading surface model. The applicability of the present model to the description of actual metal deformation behavior is verified by comparison with various cyclic loading test data.

1. Introduction

The *conventional plasticity model* (Drucker, 1988), based on the postulate that the interior of a yield surface is an elastic domain, has contributed to the prediction of the elastoplastic deformation behavior of solids such as metals, geomaterials and concretes. However, it cannot describe cyclic loading behavior and it is limited to the description of monotonic loading behavior. Various elastoplasticity models aimed at describing cyclic loading behavior have been proposed to date. A relevant description of the plastic strain rate induced by the rate of stress inside the yield surface is required for them. They are referred to as the *unconventional plasticity model* by Drucker (1988), and also called the *cyclic plasticity model* (cf. e.g. Hashiguchi, 2013b).

The *subloading surface model* (Hashiguchi, 1980, 1989, 2013a,b; Hashiguchi & Ueno, 1977) is based on the natural concept that the plastic strain rate develops continuously as the stress approaches the yield surface, renamed the *normal-yield surface*. Here, a purely elastic domain is not incorporated in the subloading surface model, although a small yield surface enclosing the purely elastic domain is incorporated and assumed to translate with a plastic deformation inducing the kinematic hardening in the other unconventional plasticity model, i.e. the so-called *cyclic kinematic hardening model*, e.g. the multi surface model (Mroz, 1967), the two surface model (Dafalias & Popov, 1975; Yoshida & Uemori, 2002) and the superposed kinematic hardening model (Chaboche, Dang-Van, & Cordier, 1979; Ohno & Wang, 1993). Note that the physical mechanism of plastic deformation would be basically different from that of the kinematic hardening. First, note that the

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plastic deformation develops in materials such as soils which are irrelevant to the kinematic hardening in general. Further, note that the purpose for the creation of unconventional elastoplasticity model is the resolution of the defect in the conventional elastoplasticity which is incapable of describing the plastic strain rate by the rate of stress inside the yield surface because of the adoption of the yield surface enclosing the purely-elastic domain. However, the defect of the conventional elastoplasticity model cannot be solved endlessly by the cyclic kinematic hardening model models inheriting the yield surface enclosing a purely-elastic domain, since they are required to incorporate smaller and smaller yield surfaces one after another endlessly depending on the stress level and the stress amplitude, like an infinity mirror, a nest of boxes, etc. in actual analysis.

First, the *subloading surface* is incorporated in this model, which always passes through the current stress point and keeps a similar shape and an orientation to the normal-yield surface. Then, the measure for the approaching degree to the normal-yield surface is given by the *normal-yield ratio*, i.e. the ratio of the size of the subloading surface to that of the normal-yield surface. The evolution rule of the normal-yield ratio is formulated such that it always approaches unity and thus the stress is always attracted to the normal-yield surface in the plastic loading process. Further, the plastic modulus is derived by substituting the evolution rule of the normal-yield ratio into the consistency condition defined by the time-differentiation of the subloading surface. Therefore, the yield judgment (i.e. the judgment as to whether the stress reaches the yield surface) is unnecessary in the loading criterion because the current stress always lies on the subloading surface which plays the role of loading surface. Here, the determination of the offset value of the yielding is unnecessary, which is influenced by an arbitrariness. Furthermore, the stress is automatically attracted to the normal-yield surface and thus an operation for pulling-back the stress to the yield surface is unnecessary in numerical calculations due to the input of finite loading increments by which the stress goes out from the yield surface in the plastic loading process. Here, the subloading surface model always fulfills not only the *continuity condition* (Hashiguchi, 1993a,b, 1977, 2000) leading to the description of a continuous response of the stress rate to the strain rate resulting in the uniqueness of solution but also the *smoothness condition* leading to a continuous tangent modulus for a continuous variation of stress state resulting in a smooth transition from the elastic to the plastic state.

The subloading surface model is regarded to be the appropriate constitutive model for a wide class of irreversible mechanical phenomena as enumerated as follows: 1) Monotonic and cyclic loading behavior can be described realistically (Hashiguchi, 2013b). 2) The overstress model is modified to be applicable to the description of elastoplastic deformation at general rate ranging from quasi-static to impact loading (Hashiguchi, 2013b); the previous overstress model is inapplicable to impact loading, resulting in the unrealistic prediction of just elastic response with an infinite strength. 3) The damage model and the phase transformation model are modified to describe softening behaviour appropriately (Hashiguchi, 2015b; Hashiguchi & Okamura, 2014). 4) The basis for constitutive modelling of fatigue phenomenon is established (Hashiguchi, 2013b); this is required to describe the accumulation of infinitesimal plastic strain rates at a low stress level, while the fatigue phenomenon cannot be described at all by conventional elastoplasticity models assuming a purely elastic domain. 5) Exact finite strain elastoplasticity (hyperelastic-based plasticity) based on the multiplicative decomposition of the deformation gradient is formulated rigorously as has been done by Hashiguchi and Yamakawa (2012) for monotonic loading behaviour and by Hashiguchi (2015a) for general loading behaviour including cyclic loading behaviour. 6) The crystal plasticity analysis, in which slip analyses in numerous slip systems are required, is realised rationally with high efficiency because a yield judgment is not required and the stress is pulled-back automatically to the yield surface (Hashiguchi, 2013a, 2015a, Hashiguchi et al., 2016), although it has been performed irrationally as the rate-dependent behaviour based on the creep model which is physically unacceptable because an unrealistic creep strain rate is always induced even during unloading after Peirce, Asaro, and Needleman, (1982, 1983). Consequently, the subloading surface model would be valid for the description of irreversible mechanical behaviour in solids for rate-independent and rate-dependent behaviours ranging from micro- to macro-levels for pressure-independent and pressure-dependent materials for finite deformation, attaining the unification of the physical relevance and the numerical efficiency. Incidentally, apart from the elastoplastic approach based on the basic postulates, i.e. the incorporations of the decomposition of strain rate into the elastic and the plastic parts and the yield surface, the constitutive relation free from these postulates has been proposed by Rajagopal and Srinivana (1998, 2009, 2015). It is capable of describing the smooth elastic-plastic transition as the subloading surface model and is of the mathematically quite simple form. It has been applied to the wide classes of materials unlimited to the steel metals, e.g. the gum metal and rocks with damage in the one-dimensional deformation process.

Complete elastoplastic constitutive equation of metals is formulated within the framework of the subloading surface model in this article, where the exact evolution rules for the normal-yield ratio and the similarity-center of the normal-yield and the subloading surfaces, i.e. *elastic-core* are incorporated, which are required to describe accurately the cyclic loading behavior. In addition, the *tangential-inelastic strain rate* induced by the deviatoric stress rate tangential to the subloading surface and the stagnation of the isotropic hardening under the cyclic loading, i.e. *cyclic isotropic hardening stagnation* are incorporated in the formulation. It differs from the former constitutive equation of metals (Hashiguchi, Ueno, & Ozaki, 2012) in various explicit formulations, i.e. the evolution rules of normal-yield ratio, the elastic-core, the kinematic hardening and the tangential-inelastic strain rate. The validity of the present model for the prediction of real deformation behavior of metals is verified by comparison with various cyclic loading test data for the proportional and non-proportional loadings.

The direct notations $\mathbf{a}\bullet\mathbf{b}$ for $a_i b_i$, $(\mathbf{AB})_{ij}$ for $A_{ir}B_{rj}$, $\mathbf{A{:}B}$ for $A_{rs}B_{rs}$, $(\boldsymbol{\Gamma}{:}\mathbf{A})_{ij}$ for $\Gamma_{ijrs}A_{rs}$, $(\mathbf{A}{:}\boldsymbol{\Gamma})_{ij}$ for $A_{rs}\Gamma_{rsij}$, $(\mathbf{a}\otimes\mathbf{b})_{ij}$ for $a_i b_j$, $(\mathbf{A}\otimes\mathbf{B})_{ijkl}$ for $A_{ij}B_{kl}$ and $(\boldsymbol{\Gamma}{:}\boldsymbol{\Xi})_{ijkl}=\Gamma_{ijrs}\Xi_{rskl}$ are used for arbitrary vectors $\mathbf{a}$, $\mathbf{b}$, second-order tensors $\mathbf{A}$, $\mathbf{B}$ and fourth-order tensor $\boldsymbol{\Gamma}$, $\boldsymbol{\Xi}$, where Einstein's summation convention is applied for a letter of the repeated suffix taking 1, 2, 3. Further, tr$\mathbf{A}$