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Original Article

A comparison between different finite elements for elastic and aero-elastic analyses

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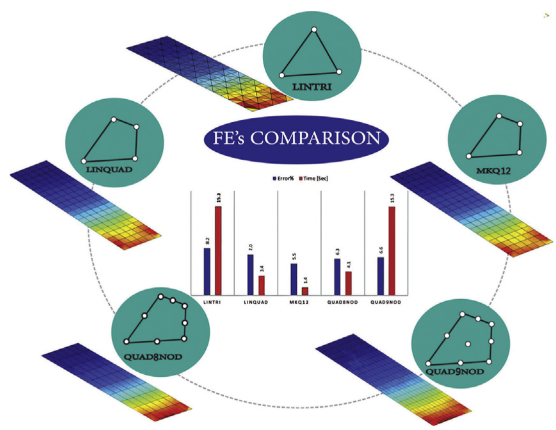
GRAPHICAL ABSTRACT



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ABSTRACT

In the present paper, a comparison between five different shell finite elements, including the Linear Triangular Element, Linear Quadrilateral Element, Linear Quadrilateral Element based on deformation modes, 8-node Quadrilateral Element, and 9-Node Quadrilateral Element was presented. The shape functions and the element equations related to each element were presented through a detailed mathematical formulation. Additionally, the Jacobian matrix for the second order derivatives was simplified and used to derive each element's strain-displacement matrix in bending. The elements were compared using carefully selected elastic and aero-elastic bench mark problems, regarding the number of elements needed to reach convergence, the resulting accuracy, and the needed computation time. The best suitable element for elastic free vibration analysis was found to be the Linear Quadrilateral Element with deformation-based shape functions, whereas the most suitable element for stress analysis was the 8-Node Quadrilateral Element, and the most suitable element for aero-elastic analysis was the 9-Node Quadrilateral Element. Although the linear triangular element was the last choice for modal and stress analyses, it establishes more accurate results in aero-elastic analyses, however, with much longer computation time. Additionally, the nine-node quadrilateral element was found to be the best choice for laminated composite plates analysis.

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Nomenclature

Symbol	
A	coefficient matrix for in-plane action
A_s	steady aerodynamic coefficient matrix in structural coordinates
A_{sd}	unsteady aerodynamic coefficient matrix in structural coordinates
A_{vlm}	steady aerodynamic coefficient matrix
B	strain-displacement matrix
d	displacement in global coordinates
D	stress-strain matrix (isotropic material properties matrix)
J	Jacobian Matrix for first order derivatives
K	stiffness matrix
M	mass matrix
N	shape function matrix
w	structural bending displacement field
W	structural bending nodal displacements
x, y, z	element local coordinates
$\boldsymbol{\delta}$	displacement vector in local coordinates
X, Y, Z	structural global coordinates
V	volume
E	elasticity modulus of the wing material
V_{inf}	flow speed
q_∞	dynamic pressure
$\boldsymbol{\sigma}$	stress
b_r	reference length (half the wing root chord)
JJ	Jacobian matrix for second order derivatives
k	reduced frequency
t	plate wing thickness
ς	wing damping ratio
ω	flutter frequency
$\boldsymbol{\varepsilon}$	strain vector
ξ, η	reference element coordinates
ρ	air density

Introduction

Numerical methods are usually the first choice for many researchers and engineers to analyze complicated systems because of their accessibility, flexibility and ability to solve complex systems. The Finite Element Method (FEM) as one of the powerful numerical methods for structural analysis comes at the top of the list of all numerical methods. As introduced in many Refs. [1–6], the method is mainly based on dividing the whole structure into a finite number of elements connected at nodes. The properties of the whole structure such as mass and stiffness, which are continuous in nature, are discretized over the elements and approximate solutions are obtained for the governing equations. The elements equations are assembled together to reach a global system of algebraic equations, which can be solved for the unknown solution variables of the structure. The accuracy of the FEM solution depends on many factors, such as the interpolation polynomials and subsequently the element shape functions, the number of degrees of freedoms selected for each element, the mesh size, and the type of element used. The model accuracy is a result of the deep understanding of the effect of each factor on the final results.

The selection of the element interpolation functions is a key factor in the accuracy of the FEM solution. For this reason, intensive researches have been made to develop new finite elements having different shapes and interpolation functions. There are numerous types of elements for different structural problems. In this paper, the main focus is on two-dimensional shell elements. Finite shell elements such as triangular elements [7–9], quadrilateral elements [10,11], higher order elements [12–17], and improved elements [18] are all tested and approved to achieve an acceptable level of accuracy. Although a vast number of elements are available in literature, researchers cannot easily figure out which element is the most suitable to select for their particular problem. The selection problem is even more difficult for engineers who are mainly interested in the application rather than the theoretical background. Additionally, the detailed mathematical formulation of some thin shell bending elements, especially the higher order ones, cannot be easily found in the literature.

Considering aero-elasticity in which the structural model is coupled to an aerodynamic model adds more complications to the problem, and makes the choice of the suitable element more challenging. Aero-elasticity is crucial for structures such as aircraft, wind turbines, and several other applications in which divergence and flutter phenomena may occur leading to catastrophic failures of the structure. Therefore, designers of these structures are constrained by the design limits and definitely need accurate FEM without being computationally expensive.

Therefore, the aim of the present work is to present a detailed mathematical formulation for different thin shell finite elements along with a complete comparison between them for specific problems in structures and aero-elasticity. The results of the selected elements are compared based on (1) solution accuracy of each element, (2) number of elements needed to achieve convergence, and (3) computational time. The comparison is for free vibration analysis, stress analysis, aero-elastic analysis, and laminated composite analysis. Five different elements are selected for the present comparison with different nature. These finite elements are

(1) Three-node linear triangular element [1] denoted as LINTRI.
(2) Four-node linear quadrilateral element [1] denoted as LINQUAD.
(3) Four-node linear quadrilateral element based on deformation modes (MKQ12 [18]).
(4) Eight-node quadrilateral element denoted as QUAD8NOD.
(5) Nine-node quadrilateral element denoted as QUAD9NOD.

These elements are selected with different nature ranging from linear to higher order, triangular to quadrilateral, and improved to regular elements to provide wide range of variety to the present comparison. All these elements are tested using bench mark problems from the literature [19,20] for elastic and aero-elastic analyses with analytical results and/or experimental measurements. The element shape functions are derived using *MATHEMATICA* [21] software and then implemented into *MATLAB* [22] codes to solve the selected problems.

The finite elements' formulation

The present finite element model is based on either the classical plate theory for metallic materials or laminated plate theory for composite materials. Both are based on the Kirchhoff assumptions which neglect the transverse shear and transverse normal effects [2].

To formulate a finite shell element there is a standard procedure that is usually followed.

(1) Start from the weak (integral) form of the governing equation.