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REVIEW ARTICLE

Solution of fractional-order differential equations based on the operational matrices of new fractional Bernstein functions

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Abstract An algorithm for approximating solutions to fractional differential equations (FDEs) in a modified new Bernstein polynomial basis is introduced. Writing $x \to x^{\alpha}$ $(0 < \alpha < 1)$ in the operational matrices of Bernstein polynomials, the fractional Bernstein polynomials are obtained and then transformed into matrix form. Furthermore, using Caputo fractional derivative, the matrix form of the fractional derivative is constructed for the fractional Bernstein matrices. We convert each term of the problem to the matrix form by means of fractional Bernstein matrices. A basic matrix equation which corresponds to a system of fractional equations is utilized, and a new system of nonlinear algebraic equations is obtained. The method is given with some priori error estimate. By using the residual correction procedure, the absolute error can be estimated. Illustrative examples are included to demonstrate the validity and applicability of the presented technique.

Contents

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1. Introduction

Fractional differential equations have several applications in mathematical physics, fluid flow, engineering and other areas of applications (Miller and Ross, 1993; Podlubny, 1999; Jafari and Momani, 2007; Daftardar and Jafari, 2007; Abdulaziz et al., 2008). In this paper, we consider the fractional differential equations (FDEs) of the form:

$$D^{\alpha}y(x) = g(x) + q(x)y(x) + z(x)(y(x))^{r}, \quad 0 \leqslant x \leqslant 1. \tag{1}$$

under the conditions

$$y(0) = \beta_1, \quad y(1) = \beta_2. \tag{2}$$

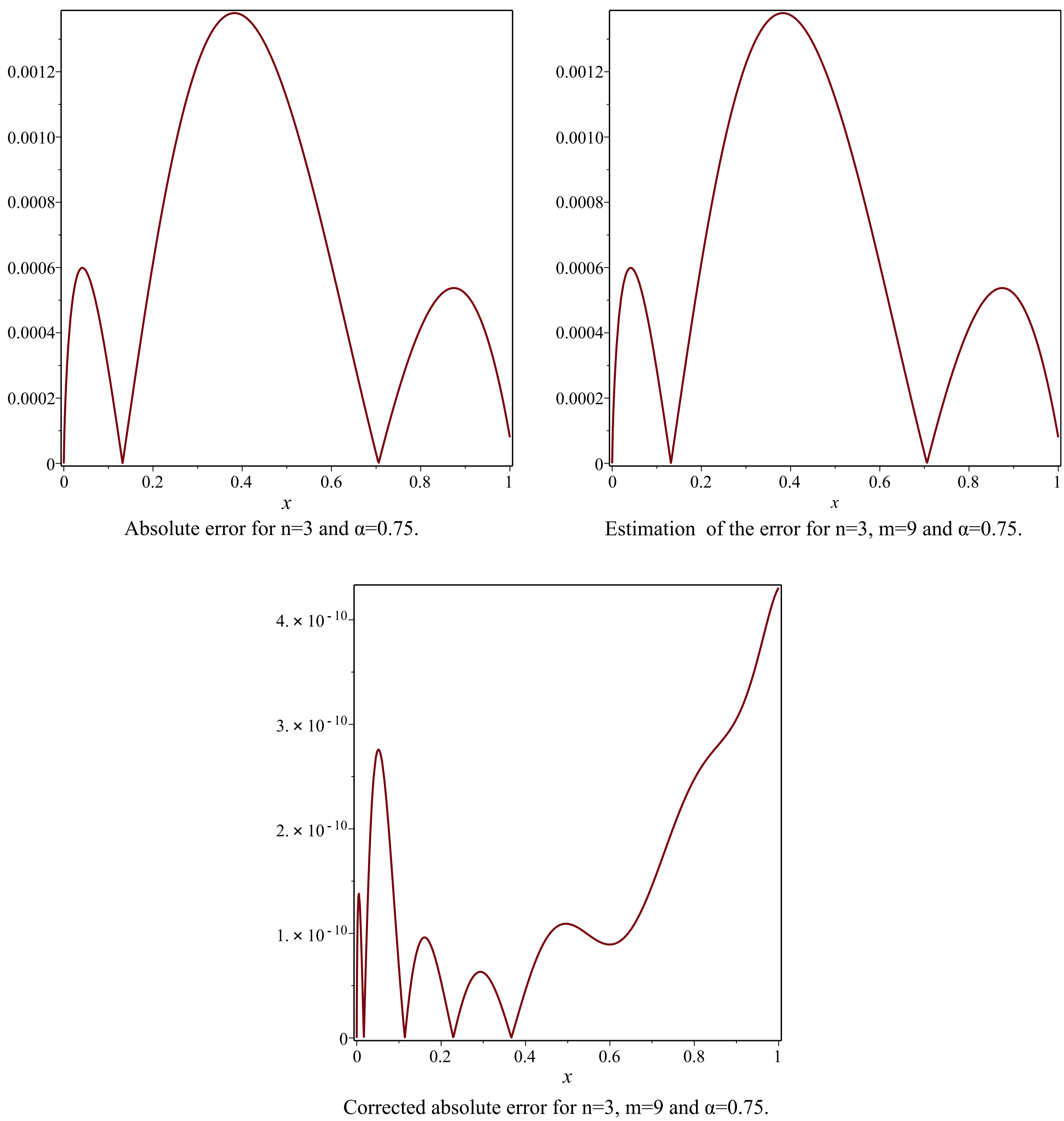

Figure 1 The absolute error, the estimated absolute error and the corrected absolute error to Example 1, for the case $n = 3, m = 9$ and $\alpha = 0.75$.