Journal of King Saud University – Science (2017) 29, 119–125

King Saud University
Journal of King Saud University – Science
www.ksu.edu.sa
www.sciencedirect.com

ORIGINAL ARTICLE

Effects on magnetic field in squeezing flow of a Casson fluid between parallel plates

Naveed Ahmed [a], Umar Khan [a], Sheikh Irfanullah Khan [a,b], Saima Bano [a], Syed Tauseef Mohyud-Din [a,*]

[a] Department of Mathematics, Faculty of Sciences, HITEC University, Taxila Cantt, Pakistan
[b] COMSATS Institute of Information Technology, University Road, Abbottabad, Pakistan

Received 5 December 2014; accepted 22 March 2015
Available online 27 March 2015

KEYWORDS

Squeezing flows;
Variation of parameters method (VPM);
Magneto hydrodynamic (MHD);
Parallel plates;
Casson fluid

Abstract Squeezing flow of an electrically conducting Casson fluid has been taken into account. The laws of conservations under the similarity transformation suggested by Wang (1976) have been used to extract a highly nonlinear ordinary differential equation governing the magneto hydrodynamic (MHD) flow. Resulting equation has been solved analytically by using the variation of parameters method (VPM). A RK-4 numerical solution has also been sought to examine the validity of analytical results. Both the solutions are found to be in an excellent agreement. Convergence of the solution is also discussed. Flow behavior under the modifying involved physical parameters is also discussed and explained in detail with the graphical aid. It is observed that magnetic field can be used as a control phenomenon in many flows as it normalizes the flow behavior. Also, squeeze number plays an important role in these types of problems and an increase in squeeze number increases the velocity profile.

1. Introduction

Many mechanical equipment work under the principle of moving pistons where two plates exhibit the squeezing movement normal to their own surfaces. Electric motors, engines and hydraulic lifters also have this clutching flow in some of their parts. Due to this practical significance squeezing flow between parallel plates has become one of the most active research fields in fluid mechanics. Its biological applications are also of equal importance. Flow inside syringes and nasogastric tubes is also a kind of squeezing flows.

Foundational work regarding squeezing flows can be named to Stefan (1874) who presented basic formulation of these types of flows under lubrication assumption. After him numbers of scientist have shown their interests toward squeezing flows and have carried out many scientific studies to understand these flows. Some of selected contributions are mentioned in forthcoming lines.

1986 Reynolds (1886) investigated the squeezing flow between elliptic plates. Archibald (1956) considered the same

* Corresponding author.
E-mail address: syedtauseefs@hotmail.com (S.T. Mohyud-Din).
Peer review under responsibility of King Saud University.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.jksus.2015.03.006
1018-3647

problem for rectangular plates. After that several attempts have been made by different researchers to understand squeezing flows in a better way (Grimm, 1976; Wolfe, 1965; Kuzma, 1968; Tichy and Winer, 1970; Jackson, 1962).

Earlier studies on squeezing flows were based on Reynolds equation whose insufficiency for some cases has been shown by Jackson (1962) and Usha and Sridharan (1996). Due to efforts of Birkhoff (1960), Yang (1958) and Wang and Watson (1979) more flexible and useful similarity transforms are now available. These similarity transforms reduce the Navier–Stokes equation into a fourth order nonlinear ordinary differential equation and have further been used in some other investigations as well (Wang, 1976; Laun et al., 1999; Hamdan and Baron, 1992; Nhan, 2000; Rashidi et al., 2008).

Flow of electrically conducting non-Newtonian fluid is a very important phenomenon as in most of the practical situations we have to deal with the flow of conducting fluid which exhibits different behaviors under the influence of magnetic forces. In these cases magneto hydro dynamic (MHD) aspect of the flow is also needed to be considered. Homotopy perturbation solution for Two-dimensional MHD squeezing flow between parallel plates has been determined by Siddiqui et al. (2008). Domairry and Aziz (2009) investigated the same problem for the flow between parallel disks. Recently, Mustafa et al. (2012) examined heat and mass transfer for squeezing flow between parallel plates using the homotopy analysis method (HAM).

In most of realistic models the fluids involved are not simple Newtonian. Complex rheological properties of non-Newtonian fluids cannot be captured by a single model. Different mathematical models have been used to study different types of non-Newtonian fluids. One of such models is known as Casson fluid model. (Mrill et al., 1965; McDonald, 1974) showed that it is the most compatible formulation to simulate blood type fluid flows. It is clear from the literature survey that the squeezing flow of a Casson fluid between the plates moving normal to their own surface is yet to be inspected.

Due to the inherent nonlinearity of the equations governing the fluid flow exact solutions are very rare. Even where they are available immense simplification assumptions have been imposed. Those overly imposed suppositions may not be used for more realistic flows. However to deal with this hurdle many analytical approximation techniques have been developed which are commonly used nowadays (Abbasbandy, 2007a; Abbasbandy, 2007b; Abdou and Soliman, 2005; Noor and Mohyud-Din, 2007; Asadullah et al., 2013; Khan et al., 2012; Ahmed et al., 2014). Variation of parameters method (VPM) is one of these recently developed analytical techniques that have been used to solve different problems (Khan et al., 2014; Noor et al., 2008; Mohyud-Din et al., 2009; Khan et al., 2014a; Khan et al., 2014b; Khan et al., 2014c; Zaidi et al., 2013).

A literature survey reveals that no attempt has ever been made to study the MHD squeezing flow of a Casson fluid. So, in this paper we have presented a comprehensive study for this problem. VPM has been applied to study the nonlinear ordinary differential equation. A numerical solution to the problem has also been sought by using the Runge Kutta order 4 method. Comparison between both the solutions shows that the results obtained by VPM are in excellent agreement with the numerical results.

2. Governing equations

We consider an incompressible flow of a Casson fluid between two parallel plates separated by a distance $z = \pm l(1 - \alpha t)^{1/2} = \pm h(t)$, where l is the initial gap between the plates (at a time $t = 0$). Additionally $\alpha > 0$ corresponds to a squeezing motion of both the plates until they touch each other at $t = 1/\alpha$, for $\alpha < 0$ the plates bear a receding motion and dilate as described in Fig. a. Rheological equation for Casson fluid is defined as under (Nadeem et al., 2012; Nadeem et al., 2013; Nadeem et al., 2014a; Nadeem et al., 2014b; Nadeem et al., 2014c; Ahmed et al., 2013; Casson, 1959; Akber and Khan, 2015; Akbar et al., 2014; Nakamura and Sawada, 1987; Nakamura and Sawada, 1988)

$$\tau_{ij} = \left\{ \begin{array}{ll} 2\left[\mu_B + \left(\frac{p_y}{2\pi}\right)\right]e_{ij}, & \pi > \pi_c \\ 2\left[\mu_B + \left(\frac{p_y}{2\pi_c}\right)\right]e_{ij}, & \pi_c > \pi \end{array} \right\} \tag{1}$$

where $\pi = e_{ij}e_{ij}$, and e_{ij} is the (i,j)th component of the deformation rate, i.e. π is the product of the component of deformation rate with itself, π_c is the critical value of the said product, μ_B is plastic dynamic viscosity of the non-Newtonian fluid and p_y is yield stress of slurry fluid.

A constant magnetic field of strength M_0 is imposed perpendicular and relatively fixed to the walls. We are also applying the following assumptions on the flow model:

(a) The effects of induced magnetic and electric fields produced due to the flow of electrically conducting fluid are negligible.
(b) No external electric field is present.

Under aforementioned constraints the conservation equations for the flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(1 + \frac{1}{\gamma}\right)\left(2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial x}\right) - \frac{\sigma\beta^2}{\rho}u, \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\left(1 + \frac{1}{\gamma}\right)\left(2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial y \partial x}\right), \end{aligned} \tag{3}$$

where u and v are the velocity components in x and y-directions respectively, p is the pressure, $\upsilon = \frac{\mu}{\rho}$ is the dynamic viscosity of the fluid (ratio of kinematic viscosity and density),

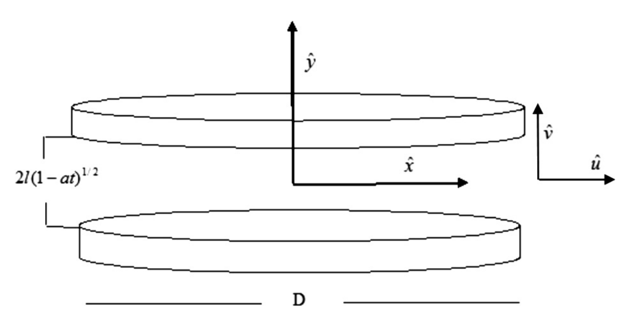

Figure a Schematic diagram for the flow problem.