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Optimal design of a cellular material encompassing negative stiffness elements for unique combinations of stiffness and elastic hysteresis

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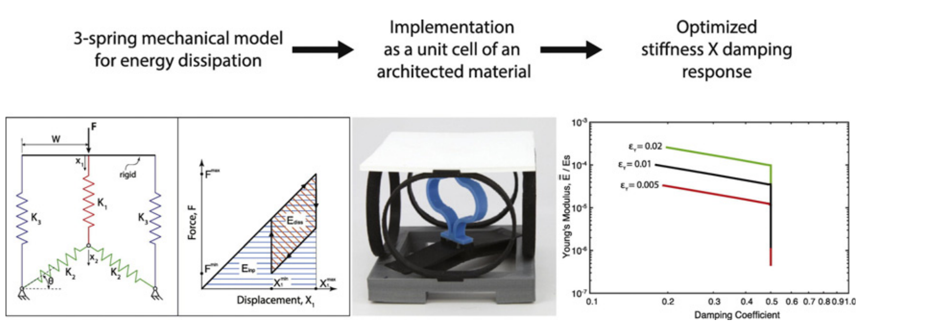
HIGHLIGHTS

- We present a 3-spring model with a negative stiffness element for energy dissipation.
- An architected material implementation of the model is designed and fabricated.
- The performance of the architected material is modeled and verified experimentally.
- The geometry of the architected material is optimized for stiffness and damping.
- This tunable stiff damper can be easily manufactured in virtually any material.

GRAPHICAL ABSTRACT



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ABSTRACT

Viscoelastic materials are commonly used to dissipate kinetic energy in case of impact and vibrations. Unfortunately, dissipating large amounts of energy in a monolithic material requires high combinations of two intrinsic properties – Young's modulus and loss factor, which are generally in conflict. This limitation can be overcome by designing cellular materials incorporating negative stiffness elements. Here we investigate a configuration comprising two positive stiffness elements and one negative stiffness element. This unit cell possesses an internal degree of freedom, which introduces hysteresis under a loading-unloading cycle, resulting in substantial energy dissipation, while maintaining stiffness. We demonstrate and optimize a simple implementation in a single material design that does not require external stabilization or pre-compression of buckled elements; these key features make it amenable to fabrication by virtually any additive manufacturing approach (from 3D printing to assembly and brazing) in a wide range of base materials (from polymers to metals). No additional intrinsic damping mechanism is required for the base material, which is assumed linear elastic. Furthermore, the architected material can be designed to be fully recoverable. When optimized, these architected materials exhibit extremely high combinations of Young's modulus and damping, far superior to those of each constituent phase.

1. Introduction

The ability to dissipate kinetic energy resulting from impact or vibration is a key requirement for a number of applications in aerospace and mechanical engineering. This energy can be dissipated in a non-recoverable way, e.g., by plastic deformation [1,2], or in a recoverable manner, generally using viscoelastic materials [3]. Recoverable behavior is generally preferred, but the amount of energy that can be dissipated using viscoelastic materials is limited by the product of Young's modulus and loss factor [4].

In recent years, a number of novel architected materials have been introduced that are based on elastic constituent phases, yet achieve hysteretic behavior under cyclic loading (and hence energy dissipation).

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Notable examples are hollow micro and nano-lattices with extremely low density which dissipate energy through internal vibration following the local buckling of the hollow bars, while maintaining linear elastic response at the constituent level up to extremely large effective strains [5,6]. Unfortunately this mechanism requires extremely low relative densities, and hence is not applicable to materials with significant strength and stiffness requirements. An alternative design is provided by entangled-wire or woven materials, which damp vibration through internal friction [7,8]. Contrary to hollow microlattices, these architected materials are generally very dense, resulting in heavier systems.

An energy dissipating mechanism that can overcome the challenges listed above is based on the snap-through buckling of elastic structural components with non-convex strain energy landscape [9,10], resulting in regimes of negative stiffness. Negative stiffness is a reversal of the usual directional relationship between force and displacement, which generally manifest itself in externally stabilized, displacement controlled structures over a range of displacements [11]: two pin-jointed springs with a non-zero initial angle, and an arch or a post-buckled beam undergoing snap-through (Fig. 1) are some examples of this concept.

The design of structures incorporating negative stiffness elements has been investigated over the past two decades, in particular with the goal of designing isolators [12–16]. Applications include seats [17, 18], earthquake protection [19] and ultra-sensitive optical devices [20]. The common configuration for a zero stiffness isolator consist of an arrangement of a positive and a negative stiffness element, combined in parallel. Tuning of the load-displacement curves for the springs can result in an effective load-displacement profile for the system featuring a flat plateau at the desired isolation force.

Recently, a number of investigators have studied simple arrangements of positive and negative stiffness elements to design structures and periodic architected materials with extreme damping. The simplest configuration consists of a stack of non-convex strain energy elements arranged in series. Notable implementations of this concept are reported in [10,9, 21,22] to design and realize cellular materials with high energy dissipation. In [23], superior damping performance was achieved by exploiting the unique post-buckling behavior of a free-end column. More complex mechanical models, consisting of series and parallel arrangements of positive and negative stiffness elements, have also been investigated [24–26]. These systems possess an internal degree of freedom that can be tuned to dramatically enhance the intrinsic loss factor of the base material under dynamic cyclic loadings. In [27], a clever implementation of a similar arrangement is used to realize an architected material that simultaneously exhibits negative stiffness and negative Poisson's ratio.

In this article, we propose and investigate a novel periodic cellular material configuration which allows independent control of stiffness and loss factor under cyclic loadings. In this novel design, each unit cell contains a particular arrangement of structural elements (henceforth called '3-spring configuration'), consisting in a positive stiffness element connected in series with a bi-stable (non-convex strain energy) element (possibly implemented by two pin-jointed springs at an angle), with the pair connected in parallel with a second positive stiffness element (Fig. 2a). This system of springs possesses an internal degree of freedom (point B), and consequently, when properly tuned, will exhibit hysteretic behavior under quasi-static cyclic loading, as schematically depicted in Fig. 2b. Clearly, the area under the hysteresis loop represents the energy dissipated in each cycle.

Although apparently similar to the models analyzed in [24–26], the proposed design is fundamentally different in that it dissipates energy via cyclic snap-through of the bi-stable element, as opposed to amplification of intrinsic damping. As discussed in detail in Section. 2.5, this results in a loss coefficient that is nearly frequency-independent and largely independent on the intrinsic material damping.

We first develop a rigorous analytical model of the 3-springs configuration, obtaining the equations that characterize its behavior under cyclic quasi-static loading. Subsequently, we adapt the model to a specific cellular material implementation, and present a simple mechanical analysis that clearly reveals the effect of the geometric parameters of the unit cell on the effective mechanical response of the material (stiffness, damping, isolation stress). The model is validated against finite elements analyses and experimental characterization, performed on a 3D printed prototype. Finally, we incorporate the validated model in an optimization framework to identify the geometric parameters that yield optimal combinations of effective Young's modulus and loss factor. Our simple implementation consists of a single material design that does not require external stabilization or pre-compression of buckled elements; these key features make it amenable to fabrication by virtually any additive manufacturing approach (from 3D printing to assembly and brazing) in a wide range of base materials (from polymers to metals) and at virtually any scale. We demonstrate that, when optimized, these architected materials exhibit extremely high combinations of Young's modulus and damping, far superior to those of the constituent material. Whereas our analysis is purely quasi-static, we show that the conclusions apply to dynamic loadings as well, as long as the frequency of oscillation is lower than the natural frequency of the internal degree of freedom, which can be tuned over wide ranges by geometrical scaling and material selection. The dynamic behavior of this system

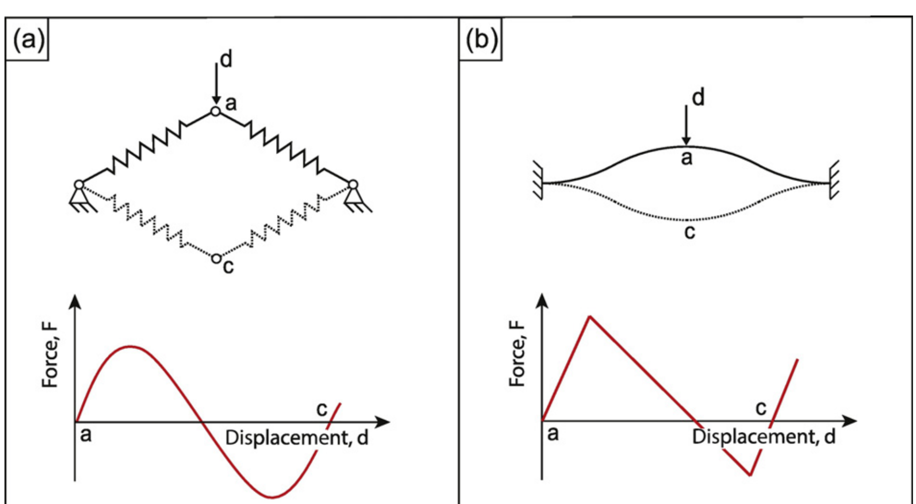

Fig. 1. Two examples of elements that can show negative stiffness, and their representative force-displacement behavior: (a) two pin joined springs arranged at an angle; (b) a sine shaped arch, with fixed end.