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# Designing biologically inspired heat conduction paths for 'volume-to-point' problems

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ABSTRACT

In this article, an evolutionary algorithm is presented for constructal optimization of high conductivity paths by mimicking the morphogenesis of leaf veins in nature. To get a physically meaningful continuous path solution, the optimization is conducted as a sequence of growth steps which begins with the heat sink and proceeds toward the whole conduction domain. The growth simulation is explored by making use of the interpolation scheme called "conductivity spreading approach" (CSA). Unlike other evolutionary algorithms, the CSA method separates growing channels from the underlying ground structure so that the growth dependency on node locations and element connections is eliminated and cooling channels are provided with more flexibility to grow toward an arbitrary direction in the conduction domain. Notably, this approach incorporates more geometry and thermal information into constructal optimization and improves the definiteness of frontiers between high and low conductivity materials. The feasibility and effectiveness of the proposed method are demonstrated in examples of 'volume-to-point' heat conduction, which forms at convergence a dendritic configuration. In addition, the influence of different design objectives, like the thermal compliance, average temperature and temperature variance, on the resulting dendritic configurations is discussed through both numerical analyses and experimental tests.

## 1. Introduction

The progress toward smaller scales in electronic equipment and the conductive cooling systems for electronic packages poses an increasingly high price on space. To sustain the projected increase in electronic power density, an efficient heat conduction path is no doubt a design priority. Related works focusing on algorithms for size and shape optimization of heat conduction paths are enormous and are continuing at an accelerated pace [1–5]. As we know, topology optimization is also an effective tool for heat conduction design, like homogenization method (HDM) [6], the solid isotropic microstructures with penalization (SIMP) method [7] and the level set (LST) method [8]. Although great contributions have been made by these methods for heat conduction design, the optimizer utilized in the SIMP or LST approaches usually leads to a pixel or node point-based representation of structural topologies. However, the pixel or node point-based solution framework is incompatible with that in CAD modelling systems where structural features are described explicitly by points, lines or curves. Therefore, it is difficult to make an accurate control of the generated structural features since no geometry information is explicitly contained in the pixel or node point-based solution, which is very important from the manufacturing perspective. Meanwhile, the constructal approach was proposed to design the optimal heat conduction path, which connects a heat generating volume to a point heat sink in terms of a tree-shaped network [9]. The constructal optimization is an assembly procedure, which starts from the smallest elemental module and proceeds toward larger size assembly units. In such assembly procedure, geometric parameters including the lengths and widths of branches [10], the angle between branches [11], as well as the branch shape [12] are optimized so as to make it possible for the thermal dynamic imperfection being distributed optimally within the conduction domain [13].

Following the early success of constructal theory, in recent years, scientists have started to consider the natural branching systems, like plant roots [14], leaf veins [15,16] and insect wings [17], as concept generators for heat conduction design. However, these works were limited to a simple mimicry of a particular branching pattern. There is little emphasis on the role of adaptive growth in forming these clever morphologies. Yet, such adaptability has profound implications for constructal optimization. An important work on transferring the growth simulation to constructal optimization was presented by Ding and Yamazaki [18], where cooling channels extended adaptively under the

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control of local temperature, which leads to an approximate uniform temperature distribution in the conduction domain. Inspired by the canalization hypothesis, Lohan et al. [19] recently proposed a bionic growth method to generate dendritic topologies for high-effective heat conduction. Although the above growth approaches are of fundamental importance, they may not give an optimum solution because no optimality conditions are imposed in the simulation. For example, the growth orientation was determined either by a pseudo-random number sequence or by a random distribution of auxin points.

To implement the growth simulation in a strict mathematical point of view, the author proposed an evolutionary algorithm by modifying the ground structure method (GSM), in which gradient-based algorithms were utilized to search for the growth orientation [20]. Although these works are straightforward to use in practice, the GSM limits the feasible design space. For example, growing elements have to be connected to the nodes of the ground structure. As a result, the growth orientation can only be selected from a few preset discrete angles, which may not include the real optimal one. To get a satisfactory design, the ground structure should be constructed as dense as possible, which limits the application to large-scale design cases. To solve this problem, we proposed a new constructal optimization method, in which the growth simulation is explored by using an interpolation scheme "conductivity spreading approach" (CSA). With such transformation, cooling channels are capable of growing toward an arbitrary direction so as to form an optimized heat conduction path.

The layout of this paper is as follows. In Section 2, the growth rule of natural branching systems is briefly summarized and transformed into a simple mathematical model. The conductivity spreading approach (CSA) and the involved finite element techniques are presented in Section 3. Then, an evolutionary algorithm is proposed for constructal optimization of high conductivity paths. Key parameters of the algorithm are discussed in Section 4. Numerical examples and experimental tests are presented in Section 5. The conclusions are given in Section 6.

## 2. Mathematical model of the growth simulation

Although the morphogenesis mechanisms and environments are different among various branching systems in nature, their diverse morphologies can be considered as a result of obeying such a common principle that they must provide the easiest way for the water and nutrient to flow through the whole network while minimize the costs of construction and maintenance of such network system under the restraints of a specific growth environment. Based on this, the implicit growth mechanism can be transformed into a group of explicit mathematical equations. The growth simulation relates to two aspects (see Fig. 1), namely: (1) searching for the optimal growth orientation; (2) searching for the optimal growth rate. Based on this, the mathematical model can be decomposed into two sub-models, where the channel orientation ($\theta$) and channel cross-sectional area ($A$) are treated as design variables, respectively.

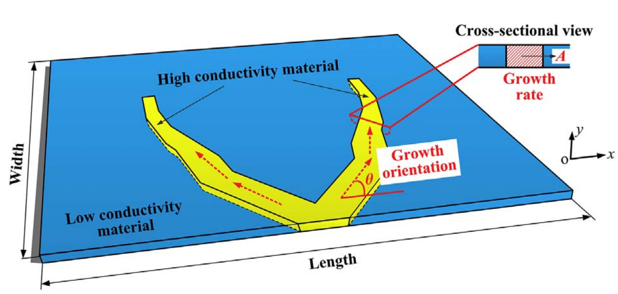


**Fig. 1.** Design variables considered in the growth simulation.

$$\begin{aligned} &\min_{\theta} \quad J\left([\theta_1^{(k)},\ \theta_2^{(k)},\ \cdots,\ \theta_{n^{(k)}}^{(k)}]\right) \\ &s.\ t. \quad 0 \le \theta_i^{(k)} < 2\pi \qquad i = 1, 2, \ldots, n^{(k)} \end{aligned} \tag{1}$$

where $J$ is the thermal design objective; $\theta_i^{(k)}$ is the orientation of the $i$-th newly generated cooling channel; $n^{(k)}$ is the number of newly generated cooling channels in the $k$-th growth step.

$$\begin{aligned} &\min_{A} \quad J\left([A_1^{(k)},\ A_2^{(k)},\ \cdots,\ A_{N^{(k)}}^{(k)}]\right) \\ &s.\ t. \quad (\Delta V)^{(k)} = \sum_{i=1}^{n^{(k)}} A_i^{(k)} \cdot L_0 + \sum_{j=1}^{N^{(k-1)}} (A_j^{(k)} - A_j^{(k-1)}) \cdot L_0 \le V_0 \\ &\qquad N^{(k-1)} = N^{(k)} - n^{(k)} \quad N^{(0)} = 0 \end{aligned} \tag{2}$$

where $J$ is the thermal design objective; $(\Delta V)^{(k)}$ is the material consumption in the $k$-th growth step and $V_0$ is the corresponding upper limit; $A_i^{(k)}$ is the cross-sectional area of the $i$-th newly generated cooling channel and $A_j^{(k)}$ is the cross-sectional area of the $j$-th already existing cooling channel; $N^{(k)}$ is the total number of cooling channels in the $k$-th growth step.

The simulation is implemented in terms of 'growth competition' and 'growth reconfiguration'. Cooling channels are first activated to participate in the growth competition for both the optimal growth orientation and the optimal growth rate. In detail, the optimal growth orientation for the newly generated cooling channels ($\theta_i^{(k)}$) are obtained by solving Eq. (1). Then, the cross-sectional area of both the newly generated ($A_i^{(k)}$) and already existing ($A_j^{(k)}$) cooling channels are updated by solving Eq. (2). After that, the thresholds for branching ($A_b^{(k)}$) and degenerating ($A_d^{(k)}$) can be calculated as follows.

$$A_b^{(k)} = 0.8 \cdot \frac{1}{N^{(k)}} \cdot \left( \sum_{i=1}^{n^{(k)}} A_i^{(k)} + \sum_{j=1}^{N^{(k-1)}} A_j^{(k)} \right) \tag{3}$$

$$A_d^{(k)} = 0.2 \cdot \frac{1}{N^{(k)}} \cdot \left( \sum_{i=1}^{n^{(k)}} A_i^{(k)} + \sum_{j=1}^{N^{(k-1)}} A_j^{(k)} \right) \tag{4}$$

Based on this, channel layouts will be reconfigured using the branching and degenerating rules like those in leaf veins. If $A_i^{(k)} > A_b^{(k)}$, the end of the $i$-th channel will be selected as a new sprouting point to generate a new branch in the next growth step, we call this operation as branching operation. If $A_i^{(k)} < A_d^{(k)}$, the $i$-th channel will be removed from the generated branch system, we call this operation as degenerating operation. The competition and reconfiguration are performed in each growth step, and repeated interactively until there are no resources available.

It can be found that the geometric information and thermal property of the cooling channels has been incorporated into the growth simulation, in which the configuration of cooling channels is generated by sequential optimization of growth orientation and growth rate according to their corresponding optimization problems. Therefore, an optimized topology of cooling channels with better cooling performance can be obtained.

## 3. Interpolation scheme: conductivity spreading approach

To make cooling channels being able to grow freely in the conduction domain, we need to separate growing elements from the underlying background structure. At this background, an interpolation scheme called "conductivity spreading approach" (CSA) is introduced in this section.

Considering a steady-state conductive heat transfer system within the domain $\Omega$, the governing equations can be expressed as

$$\nabla \cdot (k \nabla T) + f = 0 \quad in\ \Omega \tag{5}$$

where $k$ is the material thermal conductivity, $T$ is the temperature distribution over the conduction domain and $f$ is the internal volumetric heat source.