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A note on convergence of the solutions of Benjamin–Bona–Mahony type equations

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ABSTRACT

We consider the Benjamin–Bona–Mahony and the modified Benjamin–Bona–Mahony equations, which contains nonlinear dispersive effects. We prove that as the diffusion and dispersion parameters tend to zero, the solutions of these dispersive equations converge to the entropy ones of a scalar conservation law. The argument relies on deriving suitable a priori estimates together with an application of the compensated compactness method in the L^p setting.

1. Introduction

The following evolution equation

$$\partial_t u + \partial_x u^n - \partial^3_{txx} u = 0, \quad n \in \mathbb{N}, \quad n \geq 2, \tag{1.1}$$

is known as the regularized long-wave equation [1–3]. It was proposed as a model for small-amplitude long wave of water in a channel [4–6]. If $n = 2$, we recover the Benjamin–Bona–Mahony equation

$$\partial_t u + \partial_x u^2 - \partial^3_{txx} u = 0, \tag{1.2}$$

and, when $n = 3$ the modified Benjamin–Bona–Mahony one

$$\partial_t u + \partial_x u^3 - \partial^3_{txx} u = 0. \tag{1.3}$$

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(1.1) and its different variational forms were well studied both theoretically [7,8] and numerically [9,10] in the literature.

We are interested to the diffusion–dispersion limit for (1.2). Therefore, as in [11,12], we first rescale (1.2) as follows

$$\partial_t u + \partial_x u^2 - \beta\partial_{txx}^3 u = 0,$$

and consider

$$\partial_t u + \partial_x u^2 - \beta\partial_{txx}^3 u = \varepsilon\partial_{xx}^2 u. \tag{1.4}$$

As ε and β vanish we pass from (1.4) to the Burgers equation

$$\partial_t u + \partial_x u^2 = 0. \tag{1.5}$$

A first result in this direction can be found in [13], where the author used the following conserved quantity

$$t \to \int_{\mathbb{R}} \left(u^2(t,x) + \beta(\partial_x u(t,x))^2\right) dx, \tag{1.6}$$

and the assumption

$$u_0 \in L^2(\mathbb{R}) \cap L^4(\mathbb{R}), \quad \beta = \mathcal{O}\left(\varepsilon^4\right), \tag{1.7}$$

to show the convergence of the solutions of (1.4) to the distributional ones of (1.5).

The second key result on this topic can be found in [14], where the authors assume

$$u_0 \in L^2(\mathbb{R}) \cap L^4(\mathbb{R}), \quad \beta = o\left(\varepsilon^4\right), \tag{1.8}$$

and prove the convergence of solutions of (1.4) to the entropy ones of (1.5).

Generalizations of (1.4) can be found in [15–18]. In particular, in [15,18], the following equations

$$\partial_t u + \partial_x f(u) = \gamma\partial_x B(\partial_x u) + \beta\partial_{txx}^3 u - \alpha\partial_{xxxx}^4 u, \tag{1.9}$$

$$\partial_t u + \partial_x f(u) = \beta\partial_{txx}^3 u + \sum_{n=1}^{N}(-1)^{n+1}\partial_x^{2n} u \tag{1.10}$$

are studied, respectively.

About (1.9), if

$$\gamma B(\partial_x u) = \varepsilon\partial_x u, \tag{1.11}$$

(1.9) reads

$$\partial_t u + \partial_x f(u) = \varepsilon\partial_{xx}^2 u + \beta\partial_{txx}^3 u - \alpha\partial_{xxxx}^4 u. \tag{1.12}$$

Under the assumption

$$|f'(u)| \le |C|\left(1 + |u|^p\right), \quad 0 \le p < 2, \tag{1.13}$$

assuming

$$u_0 \in L^2(\mathbb{R}) \cap L^{2(p+1)}(\mathbb{R}), \quad \beta = \mathcal{O}\left(\varepsilon^{\frac{4+2p}{2-p}}\right), \quad \alpha = \mathcal{O}\left(\varepsilon^{\frac{6+p}{2-p}}\right), \tag{1.14}$$