Nonlinear Analysis: Real World Applications 40 (2018) 95–129

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

On two-dimensional incompressible magneto-micropolar system with mixed partial viscosity

Liangliang Ma

Department of Mathematics and Computer, Panzhihua University, Panzhihua 617000, PR China

ARTICLE INFO

Article history:
Received 9 December 2016
Received in revised form 28 August 2017
Accepted 30 August 2017

Keywords:
Magneto-micropolar fluid
Global well-posedness
Mixed partial viscosity
Global existence
Regularity

ABSTRACT

The magneto-micropolar fluid flows describe the motion of electrically conducting micropolar fluids in the presence of a magnetic field. The issue of whether the two-dimensional magneto-micropolar equations always possess a global (in time) classical solution can be difficult when there is only partial dissipation or no dissipation at all. In this paper, we deal with the Cauchy problem of the two-dimensional magneto-micropolar problem with mixed partial viscosity. More precisely, the global existence and regularity of classical solutions to the two-dimensional incompressible magneto-micropolar equations with mixed partial dissipation, magnetic diffusion and angular viscosity are obtained. Moreover, some conditional regularity of strong solutions is obtained for two-dimensional magneto-micropolar problem with mixed partial viscosity. This work is inspired by the recent work (Regmi and Wu, 2016) by Regmi and Wu, and our results extend their results to other mixed partial viscosities cases and global existence and regularity are established.

1. Introduction and main results

The magneto-micropolar fluid equations model the motion of electrically conducting micropolar fluids in the presence of a magnetic field, was first proposed by Galdi and Rionero in [1]. Micropolar fluids represent a class of fluids with nonsymmetric stress tensor (called polar fluids) such as fluids consisting of suspending particles, dumbbell molecules, etc. (see, e.g., [2–4]). The micropolar fluid equations ($b = 0$ in (1.1)) enable us to consider some physical phenomena that cannot be treated by the classical Navier–Stokes equations ($b = 0$ and $w = 0$ in (1.1)), such as the motion of animal blood, liquid crystals, and dilute aqueous polymer solutions [5].

Due to its important physical background, rich phenomenon, mathematical complex and challenges, the incompressible magneto-micropolar equations have been extensively studied and applied by many mathematicians, engineers and physicists.

E-mail address: mllpzh@126.com.

http://dx.doi.org/10.1016/j.nonrwa.2017.08.014
1468-1218/

The incompressible magneto-micropolar equations with full viscosity in three dimensions can be written as follows

$$\begin{cases} \partial_t u + (u\cdot\nabla)u + \nabla(p + \frac{1}{2}|b|^2) - (b\cdot\nabla)b - 2\chi\nabla\times w = (\mu+\chi)\Delta u, \\ \partial_t b + (u\cdot\nabla)b = \nu\Delta b + (b\cdot\nabla)u, \\ \partial_t w + (u\cdot\nabla)w - \kappa\nabla divw + 2\chi w - 2\chi\nabla\times u = \gamma\Delta w, \\ \nabla\cdot u = \nabla\cdot b = 0, \end{cases} \tag{1.1}$$

where $u = (u_1(x,y,t), u_2(x,y,t), u_3(x,y,t))$, $b = (b_1(x,y,t), b_2(x,y,t), b_3(x,y,t))$ and $w = (w_1(x,y,t), w_2(x,y,t), w_3(x,y,t))$ denote the velocity of the fluid, the magnetic field and the micro-rotational velocity, respectively. $p(x,y,t)$ denotes the hydrostatic pressure. μ, χ and $\frac{1}{\nu}$ are respectively, kinematic viscosity, vortex viscosity and magnetic Reynolds number. κ and γ are angular viscosities.

There is a large number of works in the literature that investigated the global regularity criteria of the strong solution of the impressible magneto-micropolar equations in three dimensions (see [6–9,11–13] and the references therein).

Suppose the magnetic field $b = 0$, then Eqs. (1.1) become the micropolar fluid equations. Dong and Zhang [14] studied the global regularity of smooth solution of the 2D incompressible micropolar fluid flows with zero angular viscosity. Chen [16] obtained the global well-posedness of the 2D incompressible micropolar fluid flows with mixed partial viscosity and angular viscosity. Łukaszewicz [15] established the global existence of weak solutions for 3D micropolar fluid equations. If $w = 0$ and $\chi = 0$, Eqs. (1.1) reduce to the magneto-hydrodynamic equations (MHD), which were studied extensively in [17–27], and the references therein. Furthermore, if $b = w = 0$ and $\chi = 0$, Eqs. (1.1) become the incompressible Navier–Stokes equations, which were studied by many scholars too, such as [28–32] and the references therein.

When $u = (u_1(x,y,t), u_2(x,y,t), 0)$, $b = (b_1(x,y,t), b_2(x,y,t), 0)$, $w = (0, 0, w(x,y,t))$, the three-dimensional magneto-micropolar equations (1.1) will be reduced to the two-dimensional magneto-micropolar equations and can be rewritten as

$$\begin{cases} \partial_t u + (u\cdot\nabla)u + \nabla(p + \frac{1}{2}|b|^2) - (b\cdot\nabla)b - 2\chi\nabla\times w = (\mu+\chi)\Delta u, \\ \partial_t b + (u\cdot\nabla)b = \nu\Delta b + (b\cdot\nabla)u, \\ \partial_t w + (u\cdot\nabla)w + 2\chi w - 2\chi\nabla\times u = \gamma\Delta w, \\ \nabla\cdot u = \nabla\cdot b = 0, \end{cases} \tag{1.2}$$

where $(x,y) \in \mathbb{R}^2$, $t > 0$, $\omega = \nabla\times u = \partial_x u_2 - \partial_y u_1$, $\nabla\times w = (\partial_y w, -\partial_x w)$.

In this paper, we consider the following two-dimensional incompressible magneto-micropolar problem

$$\begin{cases} \partial_t u_1 + (u\cdot\nabla)u_1 + \partial_x(p + \frac{1}{2}|b|^2) - (b\cdot\nabla)b_1 - 2\chi\partial_y w = \mu_{11}\partial_{xx}u_1 + \mu_{12}\partial_{yy}u_1, \\ \partial_t u_2 + (u\cdot\nabla)u_2 + \partial_y(p + \frac{1}{2}|b|^2) - (b\cdot\nabla)b_2 + 2\chi\partial_x w = \mu_{21}\partial_{xx}u_2 + \mu_{22}\partial_{yy}u_2, \\ \partial_t b + (u\cdot\nabla)b = \nu_1\partial_{xx}b + \nu_2\partial_{yy}b + (b\cdot\nabla)u, \\ \partial_t w + (u\cdot\nabla)w + 2\chi w - 2\chi\nabla\times u = \gamma_1\partial_{xx}w + \gamma_2\partial_{yy}w, \\ \nabla\cdot u = \nabla\cdot b = 0, \end{cases} \tag{1.3}$$