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Global solution to 3D spherically symmetric compressible Navier–Stokes equations with large data

Zhenhua Guo[a,*], Mei Wang[a], Yi Wang[b,c]

[a] School of Mathematics and CNS, Northwest University, Xi'an 710069, PR China
[b] CEMS, HCMS, NCMIS, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, PR China
[c] School of Mathematical Sciences, University of Chinese Academy of Sciences, Beijing, 100049, PR China

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ABSTRACT

In this paper, we proved the global well-posedness of the solution to 3D spherically symmetric, compressible and isentropic Navier–Stokes equations in the whole space with arbitrarily large initial data when the shear viscosity μ is a positive constant and the bulk viscosity $\lambda(\rho) = \rho^\beta$ with $0 \le \beta \le \gamma$ and $\gamma > 1$ being the adiabatic exponent in the γ-law pressure. First, the global classical solution is obtained away from the symmetry center $r = 0$ with arbitrarily large and non-vacuum data. In particular, it is shown that the solution will not develop the vacuum states in any finite time away from the symmetry center if the initial density does not contain vacuum states. Then the global weak solutions with the symmetry center $r = 0$ are obtained as the limit of the classical solutions in the exterior domain of a ball $B_\varepsilon(0)$ with the center at the origin and the radius $\varepsilon > 0$ when the ball shrink to the origin, that is, $\varepsilon \to 0+$, for any fixed total mass $h > 0$ defined in (3.1), and then let $h \to 0+$.

1. Introduction

The system of 3D compressible and isentropic Navier–Stokes equations reads

$$\begin{cases} \partial_t\rho + \mathrm{div}(\rho U) = 0, \\ \partial_t(\rho U) + \mathrm{div}(\rho U \otimes U) + \nabla P(\rho) = \mu\triangle U + \nabla((\mu + \lambda(\rho))\mathrm{div}U), \quad x \in \mathbb{R}^3,\ t > 0, \end{cases} \tag{1.1}$$

where $\rho(t,x) \ge 0$, $U(t,x) = (U_1, U_2, U_3)(t,x)$ represent the density and the velocity of the fluid, respectively. Here, it is assumed that

$$\mu = \text{const.} > 0, \qquad \lambda(\rho) = \rho^\beta, \quad \beta \ge 0, \tag{1.2}$$

* Corresponding author.
E-mail addresses: zhguo@nwu.edu.cn (Z. Guo), wangmei0439@163.com (M. Wang), wangyi@amss.ac.cn (Y. Wang).

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and the pressure function is given by the γ-law, that is, $P(\rho) = A\rho^\gamma$, where $\gamma > 1$ denotes the adiabatic exponent and $A > 0$ is a constant which can be normalized to be 1 for simplicity. The initial values are given by

$$(\rho, U)(t = 0, x) = (\rho_0, U_0)(x) \to (\bar{\rho}, 0), \quad \text{as} \quad |x| \to +\infty, \tag{1.3}$$

where $\bar{\rho} > 0$ is a given positive constant.

The system (1.1)–(1.2) was first proposed by Vaigant–Kazhikhov in [1] where they showed the global well-posedness of the classical solution to the 2D periodic problem with non-vacuum and arbitrarily large initial values. In particular, it is shown in [1] that the solution to the 2D periodic problem will not develop vacuum states in any finite time provided the initial density is uniformly away from vacuum. However, similar result for the 3D case is completely open due to the complicated nonlinear structures. In this paper, we consider the 3D spherically symmetric case as a first step. First, the global classical solution is obtained away from the symmetry center $r = 0$ with arbitrarily large and non-vacuum data. In particular, it is shown that the solution to 3D spherically symmetric problem will not develop vacuum states in any finite time away from the origin of the symmetry provided the initial density is away from the vacuum states. The key point in the proof lies in Lemma 2.2 for the upper and lower bounds of the density given by some well-chosen functionals. Then the global weak solutions with the symmetry center to 3D spherically symmetric Navier–Stokes equations (1.1) with large data are obtained as the limit of the classical solutions in the exterior domain of a ball $B_\varepsilon(0)$ with the center at origin and radius $\varepsilon > 0$ when the ball shrink to the origin, that is, $\varepsilon \to 0+$, for any fixed total mass $h > 0$ defined in (3.1), and then let $h \to 0+$.

There are extensive studies on global well-posedness of the compressible Navier–Stokes equations (1.1) in the case that both shear viscosity and bulk viscosity are positive constants. It is well-known that the global well-posedness theory for the one-dimension case is rather satisfactory, see [1–4] and the references therein. For multi-dimensional case, Nash, Itaya, Tani [5–7] got the local well-posedness theory of classical solutions in the absence of vacuum. The short time well-posedness of either strong or classical solutions containing vacuum states was studied recently by Cho–Kim [8] in 3D case, where a compatibility condition on the initial values is imposed when the vacuum states do occur. Then Luo [9] generalizes Cho–Kim's result [8] to the 2D case. One of the fundamental questions is whether these local (in time) solutions can be extended globally in time. The first pioneering work along this line is the well-known theory of Matsumura–Nishida [10] and after that there are some satisfactory results about the global well-posedness theory with small initial data close to a non-vacuum steady state (see [11–13]and references therein). While, the global well-posedness of classical solutions to the 3D isentropic compressible Navier–Stokes equations with small energy was proved by Huang–Li–Xin [14]. Another break-through is the global existence of weak solutions with large initial data permitting vacuum states was proved by Lions [15] with suitably large γ in the spatial dimension $N = 2, 3$. Then Jiang–Zhang [16] extend these results to the spherically symmetric case with $\gamma > 1$ and Feireisl [17] to the compressible Navier–Stokes equations with $\gamma > \frac{N}{2}$ $(N = 2, 3)$. The analysis in [15–17] allows that the initial values are arbitrarily large which may contain vacuum states. However, the regularity and uniqueness of these weak solutions are completely open in general and if the vacuum states do occur, the global well-posedness could not be expected, see [18–20] for blow-up results of classical solutions.

The case that both the shear and bulk viscosities depend on the density has also received a lot of attention recently, see [12,16,21–33] and the references therein. By some physical considerations, Liu, Xin and Yang in [31] introduced the modified compressible Navier–Stokes equations with density-dependent viscosity coefficients for isentropic fluids. In fact, as presented in [31], while deriving the compressible Navier–Stokes equations from the Boltzmann equations by the Chapman–Enskog expansions, the viscosity depends on the temperature, and correspondingly depends on the density for isentropic cases. Moreover, in geophysical flow, the viscous Saint-Venant system for the shallow water corresponds exactly to a kind of compressible Navier–Stokes equations with density-dependent viscosities, see details in [21]. It should be noted that the