



# Stationary patterns of a diffusive predator–prey model with Crowley–Martin functional response<sup>☆</sup>



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## HIGHLIGHTS

- A diffusive predator–prey system with predator interference and Neumann boundary conditions is considered.
- We show the existence and nonexistence of the nonconstant stationary solutions.
- The methods used here can be applied to other predator–prey model with predator interference.

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## ABSTRACT

A diffusive predator–prey system with predator interference and Neumann boundary conditions is considered in this paper. We derive some results on the existence and nonexistence of nonconstant stationary solutions. It is shown that there exist no nonconstant stationary solutions when the effect of the predator interference is strong or the conversion rate of the predator is large, and nonconstant stationary solutions emerge when the diffusion rate of the predator is large.

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## 1. Introduction

The interaction between the predator and prey is closely related with the functional response of the predator, which refers to the per capita feeding rate of the predator upon its prey [1,2]. In general, a diffusive predator–prey model takes the form [3]

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 \Delta u + ru \left(1 - \frac{u}{k}\right) - bp(u, v)v, & x \in \Omega, t > 0, \\ \frac{\partial v}{\partial t} = d_2 \Delta v - dv + cp(u, v)v, & x \in \Omega, t > 0, \end{cases} \quad (1.1)$$

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where  $u(x, t)$  and  $v(x, t)$  are the densities of the prey and predator at time  $t$  and location  $x$  respectively,  $d_1$ ,  $d_2$ ,  $r$ ,  $d$ ,  $k$ ,  $b$  and  $c$  are positive constants, and  $p(u, v)$  represents the functional response of the predator. If  $p(u, v)$  depends only on  $u$ , then it is referred to as the predator density-independent functional response. The predator density-independent functional responses are generally classified into four Holling types: type I–IV [4]. When  $p(u, v)$  is Holling type I functional response, that is,

$$p(u, v) = \begin{cases} u, & u < 1/\alpha, \\ 1/\alpha, & u \geq 1/\alpha, \end{cases} \quad (1.2)$$

Seo and Kot [5] found that the kinetic system of model (1.1) possesses two limit cycles and these cycles arise through global cyclic-fold bifurcations. When  $p(u, v)$  is the following Holling type II functional response

$$p(u, v) = \frac{u}{1 + \alpha u}, \quad (1.3)$$

the ODE system of model (1.1) has been investigated extensively on the aspect of the global stability and existence and uniqueness of a limit cycle [6–9]. We refer to [10–12] on the bifurcations of steady states and periodic solutions and the existence and nonexistence of nonconstant steady states for PDE system with homogeneous Neumann boundary conditions. For PDE system subject to homogeneous Dirichlet boundary conditions, Zhou and Mu [13] gave the necessary and sufficient condition for the existence of positive steady states of system (1.1). Moreover, other predator–prey models with Holling type II functional response were studied in [14–18]. When  $p(u, v)$  is Holling type III or IV functional response, the dynamics and spatiotemporal patterns of system (1.1) were investigated in [19–23] and references therein.

The above mentioned Holling type functional responses can induce different dynamical behaviors and spatiotemporal patterns, which can be used to explain the ecological complexity. However, these functional responses are all independent of the predator density, which implies that the competition among predators for food occurs only in the process of prey depletion [1]. This is not realistic sometimes and the predator interference was investigated by many researchers. For example, when  $p(u, v)$  is Holling type I functional response with predator interference, that is,

$$p(u, v) = \begin{cases} \frac{u}{1 + \beta v}, & u < 1/\alpha, \\ \frac{1}{\alpha(1 + \beta v)}, & u \geq 1/\alpha, \end{cases} \quad (1.4)$$

Seo and DeAngelis [24] studied the stability and bifurcations of equilibria for the kinetic system of model (1.1). Here  $\beta v$  models the mutual interference among predators, and if  $\beta = 0$ , Eq. (1.4) is reduced to Holling type I functional response. Similarly, the following functional response can be derived from Holling type II functional response:

$$p(u, v) = \frac{u}{1 + \alpha u + \beta v}. \quad (1.5)$$

This functional response is always referred to as the Beddington–DeAngelis (BD) functional response, which was introduced by Beddington [25] and DeAngelis et al. [26]. The dynamics of model (1.1) with BD functional response was investigated in [24,27,28]. Similarly, the following functional response, proposed by Bazykin [29] and Crowley and Martin [30],

$$p(u, v) = \frac{u}{(1 + \alpha u)(1 + \beta v)} \quad (1.6)$$

also models the predator interference, which is referred to as the Crowley–Martin (CM) functional response. For this functional response, Sambath et al. [31] studied the stability and bifurcations of the positive equilibrium of system (1.1) when the positive equilibrium is unique. Wang and Wu [32] studied a slightly

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