Nonlinear Analysis: Real World Applications 39 (2018) 93–119

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

Asymptotic behavior for cylindrically symmetric nonbarotropic flows in exterior domains with large data

Ling Wan[a], Tao Wang[b,*]

[a] School of Mathematics and Physics, China University of Geosciences, Wuhan, 430074, China
[b] School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China

ARTICLE INFO

Article history:
Received 30 September 2016
Accepted 13 June 2017
Available online 13 July 2017

Keywords:
Compressible Navier–Stokes equations
Cylindrically symmetric solutions
Asymptotic behavior
Large initial data
Unbounded domains

ABSTRACT

We study the initial–boundary value problem for the compressible Navier–Stokes equations describing the cylindrically symmetric motion of a viscous nonbarotropic fluid in the domain exterior to a ball in $\mathbb{R}^3$. The global solution is proved to exist uniquely and be asymptotically stable as time tends to infinity for large initial data. Moreover, the density and temperature are shown to be bounded from above and below uniformly in both time and space. Our analysis is based on nonlinear energy methods.

1. Introduction

The cylindrically symmetric motion of a compressible, viscous, and nonbarotropic fluid in the exterior domain $\{\mathbf{x} \in \mathbb{R}^3 : |\mathbf{x}| > a\}$ $(a > 0)$ is formulated by the compressible Navier–Stokes equations (*cf.* Landau and Lifshitz [1]):

$$\rho_t + \frac{(r\rho u)_r}{r} = 0, \tag{1.1a}$$

$$\rho(u_t + uu_r) - \frac{\rho v^2}{r} + P_r = \nu\left[\frac{(ru)_r}{r}\right]_r, \tag{1.1b}$$

$$\rho(v_t + uv_r) + \frac{\rho uv}{r} = \mu\left[\frac{(rv)_r}{r}\right]_r, \tag{1.1c}$$

$$\rho(w_t + uw_r) = \mu w_{rr} + \frac{\mu w_r}{r}, \tag{1.1d}$$

$$\rho(e_t + ue_r) + \frac{P(ru)_r}{r} = \frac{\kappa(r\theta_r)_r}{r} + \mathcal{Q}_c, \tag{1.1e}$$

* Corresponding author.
E-mail addresses: ling.wan@whu.edu.cn (L. Wan), tao.wang@whu.edu.cn (T. Wang).

http://dx.doi.org/10.1016/j.nonrwa.2017.06.006
1468-1218/

with $\nu = 2\mu + \lambda$ and

$$\mathcal{Q}_c = \nu \left[\frac{(ru)_r}{r}\right]^2 - \frac{4\mu u u_r}{r} + \mu\Big(v_r - \frac{v}{r}\Big)^2 + \mu w_r^2. \tag{1.2}$$

Here $t > 0$ is the time, $r > a$ is the radial variable, and the primary dependent variables are the density ρ, the velocity field $\mathbf{u} = (u, v, w)$, and the temperature θ. The components of velocity field $\mathbf{u} = (u, v, w)$ represent the radial, angular, and axial velocities, respectively. For ideal polytropic gases, the pressure P and the specific internal energy e are related with ρ and θ by equations of state:

$$P = R\rho\theta, \quad e = c_v\theta, \tag{1.3}$$

where $R > 0$ and $c_v > 0$ are, respectively, the gas constant and the specific heat at constant volume. The viscosity coefficients μ, λ, and the thermal conductivity coefficient κ are assumed to be positive constants (see Secchi [2] for the mathematical theory of compressible fluids with $\mu = 0$ and $\lambda > 0$).

In this paper, we establish the existence and large-time behavior of the global-in-time solutions to (1.1)–(1.3) in the unbounded domain (a, ∞) with large initial data. We shall consider the system (1.1)–(1.3) supplemented with the initial and boundary conditions:

$$(\rho, u, v, w, \theta)(0, r) = (\rho_0, u_0, v_0, w_0, \theta_0)(r), \qquad r \geq a, \tag{1.4}$$

$$(u, v, w, \theta_r)(t, a) = 0, \qquad t \geq 0. \tag{1.5}$$

The boundary conditions (1.5) are supposed to be compatible with the initial data (1.4).

Let us first mention some related results about the global solvability and large-time behavior for the nonbarotropic compressible Navier–Stokes equations with large data. Kazhikhov and Shelukhin [3] first proved the global existence and uniqueness of solutions to the compressible Navier–Stokes equations in one-dimensional bounded domains with arbitrarily large initial data. The results in [3] have been generalized to cover the spherically and cylindrically symmetric flows. In the case of spherical symmetry, Nikolaev [4] showed the existence of global-in-time (generalized) solutions in bounded annular domains, while Chen and Kratka [5] investigated the flows between a static solid core and a free boundary connected to a surrounding vacuum state. In the cylindrically symmetric case, Frid and Shelukhin [6] obtained the global solvability with large data in a bounded annular domain. Later, Hoff and Jenssen [7] proved global existence of spherically and cylindrically symmetric weak solutions with large discontinuous data in a ball. The argument in [3,6] can be also applied to the case of constant viscosity and temperature dependent thermal conductivity; see [8–11] among others. For the one-dimensional (*resp.* spherically or cylindrically symmetric) fluid in *bounded* domains, a global solution converges exponentially to a constant state as time tends to infinity, which has been established in [12] (*resp.* [13–15]). The boundedness of domains is essential in the aforementioned results.

For the cases in *unbounded* domains, the existence and uniqueness of global solutions was showed by Kazhikhov [16] for one-dimensional ideal polytropic gases with large initial data. The key point in [16] is to get positive upper and lower bounds on the density ρ and temperature θ uniformly in space. It is worth noting that, to derive the asymptotic behavior of the global-in-time solutions, one has to obtain the pointwise bounds on ρ and θ, independent of both space and time. In this direction, Jiang [17] first proved that the density ρ is bounded from below and above uniformly in both space and time by using a decent localized version of the expression on the specific volume. Based on the results in [16,17] and a time-asymptotically nonlinear stability analysis, Li and Liang [18] recently obtain the uniform-in-time upper and lower bounds on the temperature θ as well as the large-time behavior of global solutions with large data. The techniques in the works [16–18] can be employed for deducing the global solvability and asymptotic behavior of spherically symmetric solutions to the compressible Navier–Stokes equations with large initial