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Ground state sign-changing solutions for a Schrödinger–Poisson system with a critical nonlinearity in $\mathbb{R}^3$

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ABSTRACT

In this paper, we investigate the existence of ground state sign-changing solutions to a class of Schrödinger–Poisson systems

$$\begin{cases} -\triangle u + u + k(x)\phi u = \lambda f(x)u + |u|^4 u, & x \in \mathbb{R}^3, \\ -\triangle \phi = k(x)u^2, & x \in \mathbb{R}^3, \end{cases}$$

where k and f are nonnegative functions, $0 < \lambda < \lambda_1$ and λ_1 is the first eigenvalue of the problem $-\triangle u + u = \lambda f(x)u$ in $H^1(\mathbb{R}^3)$. With the help of the constraint variational method, we obtain that the Schrödinger–Poisson system possesses at least one ground state sign-changing solution for each $0 < \lambda < \lambda_1$. Moreover, we prove that its energy is strictly larger than twice that of ground state solutions. This paper can be regarded as the complementary work of Huang et al. (2013), Shuai and Wang (2015), Wang and Zhou (2015) and Zhang (2015).

1. Introduction and main results

In this paper, we are concerned with the existence of ground state sign-changing solutions of the following Schrödinger–Poisson system:

$$\begin{cases} -\triangle u + u + k(x)\phi u = \lambda f(x)u + |u|^4 u, & x \in \mathbb{R}^3, \\ -\triangle \phi = k(x)u^2, & x \in \mathbb{R}^3, \end{cases} \tag{1}$$

where $0 < \lambda < \lambda_1$ and λ_1 is the first eigenvalue of the problem $-\triangle u + u = \lambda f(x)u$ in $H^1(\mathbb{R}^3)$ and k, f are nonnegative.

System (1) stems from quantum mechanics models and semiconductor theory, and it has been studied extensively. From a physical standpoint, Schrödinger–Poisson systems describe systems of identical charged

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particles interacting each other if magnetic effects could be ignored and their solutions are standing waves. The nonlinearity models the interaction between the particles. System (1) is coupled with a Poisson equation, which implies that the potential is determined by the charge of the wave function. The term $k(x)\phi u$ considers the interaction with the electric field. For more details about the mathematical and physical background of system (1), please refer to the papers [1–3] and the references therein.

When $k(x) \equiv 0$ in system (1), it reduces to the classic semilinear elliptic problem. Bartsch, Weth and Willem [4] have obtained a ground state sign-changing solution. After that many authors are devoted to the investigations for a variety of elliptic equations on a bounded domain or the whole space. Remarkably, system (1) is nonlocal because of the presence of the term $k(x)\phi u$, which causes that the energy functional has totally different properties from the case $k(x) \equiv 0$. This phenomenon provokes some mathematical difficulties, which make the study of system (1) particularly interesting.

Schrödinger–Poisson systems have been paid much attention to various authors, especially on the existence of positive solutions, multiple solutions, ground state solutions, semiclassical states and the concentration behavior of positive solutions, see for example, [5–10] and the references therein. However, regarding the existence of sign-changing solutions for Schrödinger–Poisson systems, to the best of our knowledge, there are a few results, such as [11–22]. For example, Liu, Wang and Zhang [18] obtained infinitely many sign-changing solutions by using the method of invariant sets of descending flow when the Schrödinger–Poisson system involves a 4-superlinear and subcritical growth autonomous nonlinearity and a coercive potential function. Chen and Tang [12] studied the existence of ground state sign-changing solutions if the Schrödinger–Poisson system involves a 4-superlinear and subcritical growth non-autonomous nonlinearity and a vanish potential function.

For the following Schrödinger–Poisson system

$$\begin{cases} -\triangle u + u + k(x)\phi u = h(x,u), & x \in \mathbb{R}^3, \\ -\triangle \phi = k(x)u^2, & x \in \mathbb{R}^3, \end{cases} \tag{2}$$

when the nonlinearity satisfies 4-superlinear and subcritical growth condition on u and k is a constant, for example, if $h(u) = |u|^{p-1}u$, Wang and Zhou [21] obtained a sign-changing solution by means of a constraint variational method combining the Brouwer degree theory. Noting that the method in [21] strongly depends on the fact that the nonlinearity h is homogeneous, Shuai and Wang [20] used constraint variational methods and quantitative deformation lemma, and studied the existence and asymptotic behavior of ground state sign-changing solutions for system (2), if $h \in C^1(\mathbb{R}, \mathbb{R})$ satisfies the following conditions:

(h_1) $h(s) = o(|s|)$ as $s \to 0$;

(h_2) $\lim_{s\to+\infty} \frac{h(s)}{s^5} = 0$;

(h_3) $\lim_{s\to+\infty} \frac{H(s)}{s^4} = +\infty$, where $H(s) = \int_0^s h(t)dt$;

(h_4) $\frac{h(s)}{|s|^3}$ is an increasing function of $s \in \mathbb{R} \setminus \{0\}$.

When the nonlinearity h satisfies the critical growth condition on u, Huang, Rocha and Chen [13] and Zhang [22] studied the existence of sign-changing solutions. However, they obtained a sign-changing solution only in the case that Schrödinger–Poisson system (2) does not involve the nonlocal term, that is, $k(x) \equiv 0$. So, a natural question is whether $k \neq 0$ holds, we can also obtain the same results. Motivated by the previously mentioned works, in the present paper, we shall consider the case the nonlinearity satisfies the critical growth condition at infinity and linear growth at zero, in other words, we will investigate the existence of ground state sign-changing solutions to system (1).

In order to obtain the existence of ground state sign-changing solutions, we assume that the weight functions k, f satisfy:

(k) $k \in L^p(\mathbb{R}^3) \cap L^\infty(\mathbb{R}^3) \setminus \{0\}$ for some $p \in [2, +\infty)$ and k is nonnegative.

(f_1) $f \in L^{\frac{3}{2}}(\mathbb{R}^3) \setminus \{0\}$ is nonnegative.

(f_2) there exist $\rho > 0$ and $\alpha > 0$ such that $f(x) \geq C|x|^{-\alpha}$ for $|x| < \rho$.