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Bistability of equilibria and the 2-tori dynamics in an endogenous growth model undergoing the cusp–Hopf singularity

Giovanni Bella*, Paolo Mattana

Department of Economics and Business, University of Cagliari, Italy

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ABSTRACT

This paper studies the properties of bistability of equilibria, giving rise to periodic oscillations and 2-tori chaotic dynamics in the full three-dimensional structure of the generalized version of the Chamley (1993) endogenous growth model. This complex dynamic phenomenon reflects a particular hopf bifurcation degeneracy that originates in the neighborhood of a so-called Gavrilov–Guckenheimer singularity, with asymptotical stability properties that lead to persistent oscillations under small perturbations, until a chaos frontier is reached. As a consequence, we study all the necessary conditions, and the exact parametric configuration, that allow to locate the economy on the optimal path that avoids this undesired long run indeterminate solution.

1. Introduction

It is well established that market imperfections can undermine economic stability and lead to indeterminacy of the equilibrium. A huge bulk of literature has tackled the issue only from a local perspective, basically investigating the dynamics of the model in the vicinity of the equilibrium, by means of standard mathematical instruments (e.g., [1–5]). After the seminal works of Krugman [6] and Matsuyama [7], an increasing attention has been devoted to exploit the possibility of global indeterminacy through the occurrence of multiple equilibria or even more complex dynamics that emerge outside the small neighborhood of the steady state (e.g., [8–12]).

In a recent contribution, Bella and Mattana [13] characterize the set of conditions for the emergence of global indeterminacy of equilibria in the [14] model of endogenous growth, which originally generalizes the Lucas [15] long run mechanics resulting from the choice of investing in physical or human capital. By means of the Bogdanov–Takens bifurcation theorem, they show that a region of parameters exists at which two different equilibria coexist, one characterized by saddle-path stability, whereas the second is a non-saddle possibly surrounded by a cycle, which is in fact a low-growth trapping region.

* Corresponding author.
E-mail addresses: bella@unica.it (G. Bella), mattana@unica.it (P. Mattana).

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Our aim is twofold. First, we want to make here a step forward in the analysis and show the possibility of bistability of the steady state, and the coexistence of two different attractors, giving rise to periodic oscillations onto an invariant torus, namely a limit cycle in the three dimensional system (e.g., [16,17]). Second, since this dynamic phenomenon reflects a particular Hopf bifurcation degeneracy that originates in the neighborhood of a so-called Gavrilov–Guckenheimer singularity, we determine the asymptotical stability properties that lead to persistent oscillations, under small perturbations, until a chaos frontier is reached (e.g., [18]).[1]

The economic intuition behind this complex dynamics involves the fact that the amount of time devoted to human capital might dampen, and therefore displace, physical capital formation, thus creating instability problems. Normally, in a competitive economy, private capital owned firms adjust their level of investment, and start thus lowering the amount of employed human capital, which is now more costly, and move resources towards the physical capital sector, up to the point that its productivity starts to lower, and investment in human capital becomes again more attractive. Hence, equilibrium might perpetually oscillate between these two outcomes. In conclusion, the solution trajectories could cycle around the steady state in the short-run, or move chaotically within a bounded region, without ever attaining the former steady state again, being thus finally confined in another stationary trapping region. As a consequence, the effectiveness of any stabilizing policy action might be questioned (e.g., [22,23]). It becomes therefore interesting to study all the necessary conditions, and the exact parametric configuration, that allow to locate the economy on the optimal path that avoids this undesired long run indeterminate solution.

The paper develops as follows. In Section 2, we present the model and study the long run properties of the three dimensional vector field. In Section 3, we apply the Gavrilov–Guckenheimer theorem to infer the region of the parameter space where the equilibrium in the [14] model undergoes a cusp–Hopf bifurcation. In Section 4, an example is also provided to show the appearance of an invariant torus from the limit cycle, and how its subsequent breakdown can lead the economy to a chaotic scenario. A brief conclusive section reassesses the main findings of the paper. Appendix A provides all calculations and necessary proofs.

2. The Chamley model

The Chamley [14] model considers the following maximization problem

$$\begin{aligned} &\underset{C(t),\ u(t)}{Max} \int_0^\infty \frac{C^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt \\ &\text{s.t.} \\ \dot{K} &= AK^{1-\beta}(1-u)^\beta H^\beta - C \\ \dot{H} &= \delta H g(u, \bar{u}) \\ K(0) &= K_0 \\ H(0) &= H_0 \end{aligned} \tag{$\mathcal{P}$}$$

where C is consumption, K is physical capital, and H is human capital. Agents are endowed with a fixed amount of time, normalized to unity, which can be devoted either to the human capital sector, u, or to the physical capital sector, $1-u$, given $u \in (0,1)$. Moreover, A measures the technological level in the physical capital sector, whereas $(1-\beta) \in (0,1)$ measures the share of physical capital in production; ρ is the time preference rate, and σ is the inverse of the intertemporal elasticity of substitution.

[1] The Gavrilov–Guckenheimer singularity has found interesting applications in the field of Engineering, both in the study of stability loss in chemical reactors and the resulting chaotic patterns (e.g., [19]), or in the analysis of unwanted turbulent oscillations that emerge in coupled electrical circuits or infrastructures exposed to wind-flow extreme conditions (e.g., [20,21]).