Nonlinear Analysis: Real World Applications 39 (2018) 278–299

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

Global dynamics for a diffusive predator–prey model with prey-taxis and classical Lotka–Volterra kinetics

Tian Xiang

Institute for Mathematical Sciences, Renmin University of China, Bejing, 100872, China

ARTICLE INFO

Article history:
Received 15 February 2017
Received in revised form 24 June 2017
Accepted 8 July 2017
Available online 7 August 2017

Keywords:
Predator–prey model with prey taxis
Lotka–Volterra
Global existence
Boundedness
Asymptotics

ABSTRACT

In this paper, we use energy method to study the global dynamical properties for nonnegative solutions of the following reaction–advection–diffusion system of predator–prey model with prey-taxis and the classical Lotka–Volterra kinetics:

$$\begin{cases} u_t = d_1\Delta u - \chi\nabla\cdot(u\nabla v) - a_1u + b_1uv, & x\in\Omega, t>0,\\ v_t = d_2\Delta v + a_2v - b_2uv, & x\in\Omega, t>0 \end{cases}$$

in a bounded smooth but not necessarily *convex* domain $\Omega\subset\mathbb{R}^2$ with nonnegative initial data u_0, v_0 and homogeneous Neumann boundary data. Here, d_1, d_1, b_2 are positive, χ, a_1, b_1 are nonnegative and a_2 is allowed to be real.

It is shown that, for any regular initial data, the system has a unique global smooth solution for arbitrary size of χ, and it is uniformly bounded in time in the case of $a_2\le 0$. In the latter case, we further study its long time dynamics, which in particular imply that the prey-tactic cross-diffusion and even the linear instability of the semi-trivial constant steady states $(0, v^*)$ with $v^* > \frac{a_1}{b_1}$, $b_1>0$ and $a_2=0$ still cannot induce pattern formation. More specifically, it is shown that (u, v) converges exponentially to $(0,0)$ in the case that the net growth rate of prey is negative, i.e., $a_2<0$. In the case of $a_2=0$, we obtain the following classification for its long time behavior.

- (P1) Case I: $a_1>0, b_1=0$, then u converges exponentially to 0 and $v\to k$ in $C^2(\bar{\Omega})$, where k is a positive and finite number and it satisfies

$$(\ln k)|\Omega| = d_2\int_0^\infty\int_\Omega \frac{|\nabla v|^2}{v^2} - \frac{b_2}{a_1}\int_\Omega u_0 + \int_\Omega \ln v_0.$$

- (P2) Case II: $a_1>0, b_1>0$, then $u\to 0$ and $v\to m$ in $C^2(\bar{\Omega})$, where m is a positive and finite number and it satisfies

$$m|\Omega| = \int_\Omega v_0 + \frac{b_2}{b_1}\int_\Omega u_0 - \frac{a_1b_2}{b_1}\int_0^\infty\int_\Omega u.$$

- (P3) Case III: $a_1=0$, then $u\to(\bar{u}_0+\frac{b_1}{b_2}\bar{v}_0)$ in $C^2(\bar{\Omega})$ and $v\to 0$ exponentially, where $\bar{u}_0=\frac{1}{|\Omega|}\int_\Omega u_0$ and $\bar{v}_0=\frac{1}{|\Omega|}\int_\Omega v_0$.

The convergence properties (P1) and (P2) imply that, spatial diffusion, especially, the random movement of prey plays a role in the long time behavior

E-mail address: txiang@ruc.edu.cn.

http://dx.doi.org/10.1016/j.nonrwa.2017.07.001

and that the chemotaxis mechanism may have certain influence on its long time behavior. In particular, the long time behavior may not always be determined by its corresponding ODE system, which seems to be a rarely occurring phenomenon.

1. Introduction and sketch of the main results

Predation is one of the central features in ecological systems. Since the first ordinary differential equation of predator–prey model, known as Lotka–Volterra model, was formulated in 1920s, numerous predator–prey interaction models with or without the spatial dispersal effect have been studied. In spatial predator–prey models, besides the random movement of the predator and prey in their habitat, which is modeled by diffusion equations, it has been recognized that the spatial–temporal variations of the predator' velocity often are oriented by the prey gradient [1,2], which is a widely known chemotaxis phenomenon (One species directs its movement toward or away a higher concentration gradient of another diffusible species.)

To capture the aforementioned phenomenon mathematically, we let $u = u(x, t)$ and $v = v(x, t)$ denote the densities of predator and prey respectively at position x and time t, and let $\Omega \subset \mathbb{R}^n$ be their habitat domain and finally, we take the interaction kinetics to be the classical Lotka–Volterra nonlinearity [3] and assume the ecological system is isolated from outside so as to obtain the following initial–boundary value problem of reaction–advection–diffusion system of predator–prey model with prey-taxis:

$$\begin{cases} u_t = d_1\Delta u - \chi\nabla\cdot(u\nabla v) - a_1 u + b_1 uv & x\in\Omega, t>0,\\ v_t = d_2\Delta v + a_2 v - b_2 uv, & x\in\Omega, t>0,\\ \dfrac{\partial u}{\partial\nu} = \dfrac{\partial v}{\partial\nu} = 0, & x\in\partial\Omega, t>0,\\ u(x,0)=u_0(x)\ge,\not\equiv 0, v(x,0)=v_0(x)\ge,\not\equiv 0, & x\in\Omega. \end{cases} \tag{1.1}$$

Hereafter, $\Omega \subset \mathbb{R}^n (n \geq 1)$ is a bounded domain with a smooth boundary $\partial\Omega$ and $\frac{\partial}{\partial\nu}$ denotes the outer normal derivative; $d_1 > 0$ and $d_2 > 0$ are the diffusion rates of the predators and prey, respectively; $a_1 \geq 0$ represents the mortality rate of predators and $a_2 \in \mathbb{R}$ denotes the net growth rate of prey; the product term $+b_1uv$ measures the predation rate of predators upon preys and $b_1 \geq 0$ describes the conversion rate, similarly, $-b_2uv$ with $b_2 > 0$ describes the nontrivial interaction when applied to the prey equation; in total, the functions $-a_1u + b_1uv$ and $a_2v - b_2uv$, known as the classical Lotka–Volterra kinetics, provide the interaction kinetics between predators and prey. Finally, the advective term $-\chi\nabla \cdot (u\nabla v)$ models the directed movement that u moves toward the increasing gradient of v with $\chi \geq 0$ measuring the strength of prey taxis; this is commonly termed as chemotactic movement, a biological phenomenon whereby biological individuals orient their movement in response to some external signaling substances.

Even though reaction–diffusion systems have been extensively explored in the literature, while, in the absence of prey taxis, i.e., $\chi = 0$, we find that the reaction–diffusion system of the predator–prey model (1.1) was only recently studied in 2012 by Latos et al. [4]; therein, they first used the Neumann heat semigroup technique to show the boundedness and thus global existence of solutions in any dimension, and then they employed the classical theory of dissipative dynamical systems to study (transient and) asymptotic behavior of the solution to the prey–predator system, formulating its spatially homogeneous part as a Hamilton system; namely, its long time behavior is completely governed by its corresponding ODE:

$$\begin{cases} p_t = -a_1 p + b_1 pq & t>0,\\ q_t = +a_2 q - b_2 pq, & t>0,\\ p(0)=\bar{u}_0, \qquad q(0)=\bar{v}_0, \end{cases} \tag{1.2}$$

where here and below, for a generic function w, $\bar{w}$ denotes its spatial average $\frac{1}{|\Omega|}\int_\Omega w$.