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Optimal harvesting and stability for a prey–predator model

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This paper describes a fish prey–predator model with a new functional response. The dynamics of the system is discussed mainly from the point of view of permanence and stability. We obtain conditions that affect the persistence of the system. Local asymptotic stability of various equilibrium solutions is explored to understand the dynamics of the model system. The global asymptotic stability of positive interior equilibrium solution is established using suitable Lyapunov functional. We then examine possibilities of the existence of bionomic equilibrium. Lastly, the optimal harvesting policy is obtained by using the Pontryagin's maximum principle. The objective is to maximize the monetary social benefit as well as conservation of the ecosystem.

1. Introduction and mathematical model

Predator–prey dynamics are usually represented by a functional response, which is the amount of prey eaten per predator and per unit of time. This functional is a proxy of the flux of matter from one trophic level to another as it determines the transfer of biomass in the food chain [1]. Typically, a predator–prey model focuses on interactions between two species taking into account some aspects that are considered nodal to explain the dynamics. These interactions depend on the nature of the studied species [2–4]. Recently, in [5], authors proposed a new response functional in order to explain the influence of changing water level fluctuations in an artificial lake on fish predator–prey dynamics. In the studied lake, two interdependent species are considered; the pike (brochet in French) which is the most important predator and the roach (gardon in French) which is the prey. This response functional is based on the following general considerations. When a predator attacks a prey, it has access to a certain quantity of food depending on the water level. When the water level is low, during the autumn, the predator is more in contact with the prey, and the predation increases. Conversely, when the water level is high, in the spring, it is more difficult for the predator to find a prey and the predation decreases. It is assumed that the accessibility function $b(t)$ for

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the prey is continuous and 1-periodic, the minimum value b_1 is reached in spring and the maximum value b_2 is attained during autumn. The predator needs a quantity γ as food, but it has access to a quantity

$$g(x,y)=\frac{b(t)x}{y+D},$$

which depends on the water level, where D measures other causes of mortality outside of predation. Thus, if

$$g(x,y)\geq\gamma,$$

then the predator will be satisfied with the quantity γ for his food. Otherwise, if

$$g(x,y)<\gamma,$$

the predator will content himself with

$$g(x,y)=\frac{b(t)x}{y+D}.$$

To summarize, the quantity of food received per predator and per unit of time is

$$\min\left(\frac{b(t)x}{y+D},\gamma\right). \tag{1}$$

The authors in [5] studied the following non-autonomous prey–predator model

$$\begin{cases}\dot{x}=ax(t)\left(1-\dfrac{x(t)}{K}\right)-\min\left(\dfrac{b(t)x(t)}{y(t)+D},\gamma\right)y(t),\\ \dot{y}=-dy(t)+e\min\left(\dfrac{b(t)x(t)}{y(t)+D},\gamma\right)y(t).\end{cases} \tag{2}$$

The constants mentioned above are all positive. The prey grow logistically with carrying capacity K and intrinsic growth rate a. By using Gaines and Mawhin's continuation theorem of coincidence degree theory [6], the authors have established sufficient conditions for the existence of positive periodic solutions of the prey–predator system (2). Such a solution describes an equilibrium situation consistent with the variability of environmental conditions, such that both populations survive. The trajectories in the phase plane of these solutions of the nonautonomous system take the place of the equilibria points of the autonomous system. In the numerical simulations given in [5], the periodic predation rate function $b(t)=b(1+0.5cos(2\pi t))$ is used, for more details, see [5,7–9].

In the present work, we focus on the autonomous case and use as predation rate, the mean function $b=\int_0^1 b(t)dt$. Moreover, to investigate the effects of harvesting on the prey–predator ecosystem, we incorporate and extend the work done by [5]. We aim to obtain some results which are theoretically beneficial to maintaining the sustainable development of the prey–predator system as well as keeping the economic interest of harvesting at an ideal level. Therefore, we study the following prey–predator model:

$$\begin{cases}\dot{x}=ax(t)\left(1-\dfrac{x(t)}{K}\right)-\min\left(\dfrac{bx(t)}{y(t)+D},\gamma\right)y(t)-qEx(t):=F_1(x,y),\\ \dot{y}=-dy(t)+e\min\left(\dfrac{bx(t)}{y(t)+D},\gamma\right)y(t):=F_2(x,y),\end{cases} \tag{3}$$

where q is the catchability coefficient of the prey species and E denotes the effort devoted to the harvesting.

The present article is organized as follows: In Section 2, we focus on the dynamics of the system (3), specifically, we establish sufficient criteria for the boundedness, permanence, and predator extinction. The local and the global stability of the dynamical system for the model are studied in Section 3. In Section 4, the existence of a bionomic equilibrium is investigated. The optimal harvesting policy is studied with the help of Pontryagin's maximum principle in Section 5. Some numerical examples are taken up to illustrate the results. Brief concluding remarks are given in Section 6 to close this work.