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On steady flow of non-Newtonian fluids with frictional boundary conditions in reflexive Orlicz spaces

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ABSTRACT

A stationary viscous incompressible non-Newtonian fluid flow problem is studied with a non-polynomial growth of the extra (viscous) part of the Cauchy stress tensor together with a multivalued nonmonotone frictional boundary condition described by the Clarke subdifferential. We provide an abstract result on existence of solution to a subdifferential operator inclusion and a hemivariational inequality in the reflexive Orlicz–Sobolev space setting modeling the flow phenomenon. We establish the existence result, and under additional conditions, also uniqueness of a weak solution in the Orlicz–Sobolev space to the flow problem.

1. Introduction

In this paper we investigate the steady generalized Navier–Stokes equation modeling a flow for incompressible fluid in a regular domain Ω in $\mathbb{R}^d$, $d = 2, 3$ of the form

$$-\operatorname{div} \mathbf{S} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nabla \pi = \boldsymbol{f}, \quad \operatorname{div} \boldsymbol{u} = 0 \text{ in } \Omega. \tag{1}$$

The extra stress tensor $\mathbf{S}$ satisfies a general nonlinear constitutive law in terms of the symmetric gradient $\boldsymbol{\varepsilon}(\boldsymbol{u}) = \frac{1}{2}(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^\top)$ formulated via an Orlicz function Φ. The prototype of such law given by $\Phi(t) = t^p$ with $1 < p < \infty$ leads to the power-law model

$$\mathbf{S}(\boldsymbol{\varepsilon}(\boldsymbol{u})) = (1 + |\boldsymbol{\varepsilon}(\boldsymbol{u})|)^{p-2}\boldsymbol{\varepsilon}(\boldsymbol{u})$$

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which is common to model non-Newtonian fluids. The problem (1) is supplemented by nonstandard boundary conditions of the form

$$u_\nu = 0, \quad -\mathbf{S}_\tau \in \partial j_\tau(\boldsymbol{u}_\tau) \text{ on } \Gamma_0, \tag{2}$$

where Γ_0 represents a part of the boundary $\partial\Omega$, u_ν, $\mathbf{S}_\tau$ and $\boldsymbol{u}_\tau$ denote the normal velocity, and the tangential components of the stress and of the velocity, respectively. Here, ∂j_τ stands for the subgradient in the sense of Clarke of a given locally Lipschitz function j_τ, for concrete examples of the function j_τ, see Section 5. The first condition in (2) is the impermeability (no leakage) condition which means that there is no flux through Γ_0, the second one represents a nonlinear frictional (possible multivalued) slip of the fluid on this part of the boundary. These nonstandard conditions (2) arise in the problem of motion of a fluid through a channel or a tube (see [1–5] for details). Because of the special form of boundary conditions, the weak formulation of problem (1), (2) has the form of a hemivariational inequality in the Orlicz space involving nondifferentiable and nonconvex functionals on the boundary.

The main goal of this paper is to prove the existence and uniqueness of solution in the reflexive Orlicz–Sobolev space to an operator inclusion with the Clarke subgradient operator and the Navier–Stokes type operator which is associated with the aforementioned hemivariational inequality.

Our approach is based on the compactness of a trace operator in the Orlicz–Sobolev space [6], coercivity and pseudomonotonicity of the Navier–Stokes type operator [5], surjectivity results for pseudomonotone maps [7] and the result on an integral representation of the Clarke subdifferential of locally Lipschitz integral functionals defined on the Orlicz space (see [8, Theorem 4.3] and [9, Corollary 6.2]). In the treatment of this topic, we successfully use some techniques from the theory of hemivariational inequalities in Sobolev spaces of Panagiotopoulos [10,11], Naniewicz and Panagiotopoulos [12], and Migórski et al. [13]. The frictional contact boundary conditions for steady/unsteady Newtonian or non-Newtonian fluid flows in Sobolev spaces setting have been studied, for instance, in [5,14–20].

The novelty of the paper consists in the fact that the constitutive relation for the stress tensor has a non-polynomial growth and it does not have an explicit form in the context of the frictional contact law described by Clarke subgradient. The frictional contact boundary condition is also established for functions of a non-polynomial growth. Besides, to the best of our knowledge, there are no results on the existence and uniqueness of solution to hemivariational inequalities in the Orlicz–Sobolev space for contact problems arising in mechanics, including Newtonian or non-Newtonian fluid flow problems.

Mathematical analysis of non-Newtonian fluids can be found in [21–26] in the stationary case and in [27–36] for the evolutionary case. Note that Problem 5.1 has been studied in [5] in the 2D setting for a particular geometry of the domain in the context of the lubrication theory and with a polynomial growth for the stress deviator. Theorem 5.5 enhances the conclusion of [5] in the 2D setting. To obtain integrability of the convective term, we assume different values of p depending on the dimension: $p \geq \frac{2d}{d+1}$ for $d = 2$, and $p \geq \frac{3d}{d+2}$ for $d = 3$. The improvement in the case $d = 2$ is commented in Remark A.4.

The content of the paper is as follows. In Section 2 we recall some notation and present some auxiliary material. In Section 3 we establish the existence and uniqueness of solution to a subdifferential operator inclusion in Orlicz–Sobolev spaces. In Section 4 we specialize our existence and uniqueness results in study of a stationary hemivariational inequality in Orlicz–Sobolev spaces. The formulation of the boundary value problem for the stationary flow of an isotropic incompressible viscous non-Newtonian fluid without the rheology and with a multivalued nonmonotone subdifferential frictional boundary condition as a hemivariational inequality is given in Section 5. In that section we deliver the results on the existence and uniqueness of the weak solution to the hemivariational inequality. In the last section we give remarks and auxiliary results.