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Global existence of solutions to system of polytropic gas dynamics with friction

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ABSTRACT

In this paper, we study the existence of global entropy solutions for the Cauchy problem of system of polytropic gas dynamics in a divergent nozzle with a friction (1.1) with bounded L^∞ initial data (1.2).

1. Introduction

In the present paper, we study the global existence of weak solutions for the following system of polytropic gas dynamics in a divergent nozzle with a friction

$$\begin{cases} \rho_t + (\rho u)_x = -\dfrac{a'(x)}{a(x)}\rho u \\ (\rho u)_t + (\rho u^2 + P(\rho))_x = -\dfrac{a'(x)}{a(x)}\rho u^2 - \alpha \rho u|u|, \end{cases} \tag{1.1}$$

with bounded L^∞ initial data

$$(\rho(x,0), u(x,0)) = (\rho_0(x), u_0(x)), \quad \rho_0(x) \geq 0, \tag{1.2}$$

where ρ is the density of gas, u the velocity, $P(\rho) = \frac{1}{\gamma}\rho^\gamma$, the pressure, the adiabatic exponent $\gamma > 1$, $a(x)$ is a slowly variable cross section area at x in the nozzle and α denotes a friction constant. Such a friction, the physical phenomena called "choking or choked flow" occurs in the nozzle (see [1,2] for the details). When

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$a'(x) = 0$, (1.1) is a shallow-water model describing the vertical depth ρ and mean velocity u, where $\alpha\rho|u|u$ corresponds physically to a friction term and α is a nonnegative constant. When $\alpha = 0$, i.e., the nozzle flow without friction, system (1.1) was studied in [3–5] for the nozzle flow with the monotone cross section $a'(x) \geq 0$; in [6] for the Laval nozzle and in [7] for the transonic flow in a nozzle with a general cross section area. The existence of solutions for the system with the ordinary friction $\alpha\rho u$ was studied in [8]. The same inhomogeneous problem was also studied in the papers [9–13].

The existence of global entropy solutions for the above hyperbolic system (1.1) was obtained in [2] for the usual gases $1 < \gamma \leq \frac{5}{3}$, provided that the initial data are bounded and satisfy $z(\rho_0(x), u_0(x)) \leq 0$, where the Godunov scheme and the fractional step procedure was used to construct approximate solutions, and the compensated compactness was used to prove their convergence. In this paper, we use the maximum principle to obtain a simple proof of the uniformly bounded L^∞ estimate $z(\rho^{\delta,\varepsilon}, u^{\delta,\varepsilon}) \leq 0, w(\rho^{\delta,\varepsilon}, u^{\delta,\varepsilon}) \leq M$, where z, w are Riemann invariants of (1.1), for the ε-viscosity and δ-flux-approximation solutions of (1.1) when the adiabatic exponent $\gamma > 1$ and also use the compensated compactness theory [14–18] to prove the convergence of $(\rho^{\delta,\varepsilon}, u^{\delta,\varepsilon})$, and the global existence of entropy solutions for the Cauchy problem (1.1)–(1.2).

2. A priori L^∞ estimate and existence results

When we use the maximum principle to obtain the uniform L^∞ estimate of approximated solutions of the Cauchy problem (1.1)–(1.2), a basic difficulty is the super-linear source terms on the right hand-side of (1.1). To overcome this difficulty, we still use the technique of the δ-flux-approximation given in [19] coupled with the approximation in the source terms. In a similar way like [5,19] , we introduce the sequence of systems

$$\begin{cases} \rho_t + (-2\delta u + \rho u)_x = A(x)(\rho - 2\delta)u \\ (\rho u)_t + (\rho u^2 - \delta u^2 + P_1(\rho, \delta))_x = A(x)(\rho - 2\delta)u^2 - \alpha\rho u|u| \end{cases} \tag{2.1}$$

to approximate system (1.1), where $A(x) = -\frac{a'(x)}{a(x)}, \delta > 0$ denotes a perturbation constant and the perturbation pressure

$$P_1(\rho, \delta) = \int_{2\delta}^{\rho} \frac{t - 2\delta}{t} P'(t)dt. \tag{2.2}$$

However, the friction term $\alpha\rho u|u|$ on the right-hand side of (2.1) will bring us some new technical difficulties.

Now we add the viscosity terms to the right-hand side of (2.1) to obtain the following parabolic system

$$\begin{cases} \rho_t + ((\rho - 2\delta)u)_x = A(x)(\rho - 2\delta)u + \varepsilon\rho_{xx} \\ \\ (\rho u)_t + (\rho u^2 - \delta u^2 + P_1(\rho, \delta))_x = A(x)(\rho - 2\delta)u^2 - \alpha\rho u|u| + \varepsilon(\rho u)_{xx}, \end{cases} \tag{2.3}$$

with initial data

$$(\rho^{\delta,\varepsilon}(x, 0), u^{\delta,\varepsilon}(x, 0)) = (\rho_0(x) + 2\delta, u_0(x) + \delta L), \tag{2.4}$$

where $(\rho_0(x), u_0(x))$ are given in (1.2) and L is a suitable large positive constant.

Applying for the maximum principle to the first equation in (2.3), we can easily obtain the following positive lower estimate

$$\rho^{\delta,\varepsilon}(x, t) \geq 2\delta,$$

which guarantees the regularity of the flux functions in (2.3).

By simple calculations, two eigenvalues of system (1.1) are

$$\lambda_1 = u - \sqrt{P'(\rho)}, \qquad \lambda_2 = u + \sqrt{P'(\rho)} \tag{2.5}$$