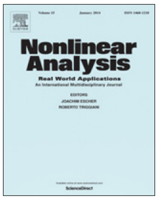
Nonlinear Analysis: Real World Applications 39 (2018) 492–519

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa



Sliding motion and global dynamics of a Filippov fire-blight model with economic thresholds

Can Chen[a,b], Yanmei Kang[b], Robert Smith?[c,*]

[a] *Department of Mathematics and Physics, College of Science, Zhengzhou University of Aeronautics, Zhengzhou 450015, PR China*
[b] *Department of Applied Mathematics, Xi'an Jiaotong University, Xi'an 710049, PR China*
[c] *Department of Mathematics and Faculty of Medicine, The University of Ottawa, 585 King Edward Ave, Ottawa, ON K1N 6N5, Canada*

ARTICLE INFO

Article history:
Received 26 October 2016
Received in revised form 8 June 2017
Accepted 3 August 2017
Available online 29 August 2017

Keywords:
Fire blight
Filippov system
Economic threshold
Threshold policy
Equilibrium
Sliding mode

ABSTRACT

Cutting off infected branches has always been an effective method for removing fire-blight infection in an orchard. We introduce a Filippov fire-blight model with a threshold policy: cutting off infected branches and replanting susceptible trees. The dynamics of the proposed piecewise smooth model are described by differential equations with discontinuous right-hand sides. For each susceptible threshold value S_T, we investigate the global dynamical behaviour of the Filippov system, including the existence of all the possible equilibria, their stability and sliding-mode dynamics, as we vary the infected threshold level I_T. Our results show that model solutions ultimately approach the equilibrium that lies in the region above I_T or below I_T or on $I = I_T$, or the equilibrium $E_T = (S_T, I_T)$ on the surface of discontinuity. Furthermore, control strategies should be taken when the solution of this system approaches the equilibrium that lies in the region above I_T. The findings indicate that proper choice of susceptible and infected threshold levels can either preclude an outbreak of fire blight or lead the number of infected trees to a desired level.

1. Introduction

Fire blight is one of the major threats to fruit-bearing trees, primarily apple, pear and other members of the Rosaceae family, due to the fact that it can destroy an entire orchard in a single growing season [1–3]. The infection is transmitted by gram-negative bacteria, *Erwinia amylovora*, which is capable of infecting blossoms, vegetative shoots, woody tissues, rootstock crowns and fruits of the trees [4,5]. The total economic loss of fire blight is not always easy to appreciate, as it is an erratic disease, but severe outbreaks can lead to millions of dollars of production and tree losses [6]. In the USA alone, it has been reported that the annual economic loss is approximately $100 million [7,8]. Similarly, in Europe, significant economic losses have been

---

[*] Corresponding author.
E-mail addresses: can3111054023@stu.xjtu.edu.cn (C. Chen), ymkang@mail.xjtu.edu.cn (Y. Kang), rsmith43@uottawa.ca (R. Smith?).

http://dx.doi.org/10.1016/j.nonrwa.2017.08.002

reported; e.g., in Switzerland, a major outbreak of fire blight occurred between 1997 and 2000, resulting in the loss of $9 million within this period [9]. Furthermore, the worldwide economic importance due to this disease is likely to increase [10].

Currently, even though there is no cure for fire blight, preventative strategies have been implemented to reduce the spread of fire-blight infection, such as pruning and removal of diseased plant parts. Furthermore, it has been recognized that cutting off infected branches is an effective method of removing fire-blight infection, because it can disrupt the equilibrium between vegetative and reproductive growth [11,12]. Two sets of experiments were conducted during 1999 to 2001 in Israel to evaluate the efficacy of pruning infected pear tissues to combat fire blight. They found that if pruning was carried out when the trees were dormant (in December), then none of these plants had a severely infected canopy the following spring [13]. Nevertheless, the loss of fruit production can be economically devastating for growers, even if the disease does not kill the tree. In reality, complete eradication of the infected trees is generally not possible, nor is it economically desirable. Therefore an efficient control strategy is needed to avoid overpruning and reduce economic losses.

Mathematical models can be a useful tool for designing strategies to control the spread of plant diseases and determining their efficacy, especially in the absence of an effective treatment [14]. Many different types of mathematical models on plant diseases have been proposed [15,16], including ordinary differential equation models [17,18] and impulsive differential equation models [19,20]. A combination of an epidemiological model, together with the analysis of evolutionary stable strategies, was used to analyse the effectiveness of continuous control measures for combating vegetatively propagated plant diseases [17]. Meng et al. [19] constructed plant-disease models with continuous and impulsive cultural control strategies to investigate how to control plant-disease transmission by replanting of healthy plants and removal of infected trees. Tang et al. [20] first developed a plant-disease model with pulse replanting and roguing strategies at fixed moments, then formulated a state-dependent impulsive model by implementing a cultural control strategy only when the number of infected plants reaches an economic threshold value.

However, in these plant-disease models, there exist some disadvantages. On the one hand, if control strategies occur continuously or impulsively at fixed moments, regardless of whether the number of infected trees reaches the economic threshold or not, this will consume a huge amount of economic damage and labour costs, because it is not necessary to implement the control strategy when the number of infected plants is not relatively high. On the other hand, in the state-dependent impulsive plant disease models, once the number of infected trees reaches the economic threshold, the growers would theoretically implement the control strategy instantaneously and reduce it below the economic threshold at that precise moment, which seems unrealistic.

Consequently, a more realistic threshold policy is required to provide useful information in fire-blight management strategies, so that the economic damage can be reduced to a minimum level. Therefore, by incorporating non-instantaneous control with the threshold policy, the spread of fire blight can be described by nonlinear ordinary differential equations with discontinuous right-hand sides, called Filippov systems [21,22]. Although Filippov systems have been used to investigate many infectious diseases [23–26], very little is known about the effects of the discontinuous control functions on the dynamics of fire-blight. Thus our main purpose is to construct a Filippov fire-blight model by considering cutting off infected branches and replanting susceptible trees. Then, by applying the theory of Filippov systems to the proposed model, we aim to establish conditions under which the growers can achieve minimize economic losses and maximize returns.

The rest of this paper is structured as follows. In Section 2, we propose a Filippov fire-blight model incorporating cutting off infected branches and replanting susceptible trees. The dynamical behaviour of the proposed Filippov system, including the existence of all the possible equilibria, their stability and sliding-mode dynamics, is investigated by varying the infected and susceptible threshold values in Sections 3–6. Finally, we present a discussion and biological conclusion on the results of this work in Section 7.