Nonlinear Analysis: Real World Applications 37 (2017) 71–93

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

Existence of solution to a model for gas transportation networks on non-flat topography

Alfredo Bermúdez[a,*], Julio González-Díaz[b], Francisco J. González-Diéguez[a]

[a] *Dpto. de Matemática Aplicada, Universidade de Santiago de Compostela, Campus Sur s/n, 15782 Santiago de Compostela, Spain*

[b] *Dpto. de Estadística e Investigación Operativa, Universidade de Santiago de Compostela, Campus Sur s/n, 15782 Santiago de Compostela, Spain*

ARTICLE INFO

Article history:
Received 19 February 2016
Received in revised form 18 October 2016
Accepted 7 February 2017

Keywords:
Gas transportation networks
Existence of solution
Convex analysis
Fixed-point theorem

ABSTRACT

In this paper we prove the existence of solution to a mathematical model for gas transportation networks on non-flat topography. Firstly, the network topology is represented by a directed graph and then a nonlinear system of numerical equations is introduced whose unknowns are the pressures at the nodes and the mass flow rates at the edges of the graph. This system is written in a compact vector form in terms of the vector of the square pressures at the nodes and then an existence result is proved under some simplifying assumptions. The proof is done in two steps: the first one uses convex analysis tools and the second one the Brouwer fixed-point theorem.

1. Introduction

Due to their nature, gas transmission networks occupy vast extensions of land which can be measured in thousands of kilometers. They are generally managed from a control center by the Technical System Manager (TSM) based on the data they receive from the different elements that form the network, namely, compression stations, pressure control valves, flow control valves, closing valves, regasification plants, international connections, underground storages and deposit fields. A gas transmission network is defined by its topology and its elements. More specifically, the gas pipeline connections and geometrical properties as length, diameter, roughness, and geographic coordinates of pipes and nodes.

Mathematical modeling of gas flow in pipelines is an important subject in planning and operating gas transportation networks (see reference books, [1,2]). Some recent papers have been devoted to the transient

* Corresponding author.
E-mail addresses: alfredo.bermudez@usc.es (A. Bermúdez), julio.gonzalez@usc.es (J. González-Díaz), franciscojose.gonzalez@usc.es (F.J. González-Diéguez).

http://dx.doi.org/10.1016/j.nonrwa.2017.02.003
1468-1218/

case (see [3,4]). Usually, they assume isothermal or isentropic flow (see [5]) but in real networks neither temperature nor entropy remain constant because, first, there is heat exchange with the environment (see [6,7]) and second there is dissipation in the boundary layer near the wall of the pipelines due to viscous friction. These features complicate the model because they lead to two respective source terms in the physical balance laws.

However, most papers and computer programs on the subject deal with the case of steady-state because, based on simplified steady state models, network optimization problems can be stated and solved even for large networks. Currently, in most networks the first aim is to meet all consumer demands, the so-called *security of supply*. However, compression stations and regasification plants use themselves the gas as an internal power source, thus leading to operation costs. In order to minimize these costs it is necessary to pay particular attention to the way in which the network is managed. Mathematical optimization theory is an important tool to handle this problem (see [8–12]). Nevertheless, the analysis of these optimization problems is beyond the scope of this paper which is rather focused in the network simulation problem when the flow inputs and outputs are given and the operating parameters of compressors (compression ratio or pressure jump) are prescribed; more specifically, we deal with the existence of solution of a particular but frequently used network mathematical model.

Despite their practical importance, to the best of the authors' knowledge, the proof of the existence of a solution to these simplified models has been only done for the case of flat topography (see [13, Corollary 2]) and strictly speaking for a network involving only pipes, neither compressors nor valves are considered. Thus, the nontrivial extension of this result to the full more general model with non-flat topography remains an open problem and it is the main goal of the present research. It is worth emphasizing that the fact that nodes can have different heights has an extremely important influence on the behavior of gas networks. For instance, an upwinding node may have a lower pressure than a downwinding one if the level of the former is higher than the level of the latter. Of course this would never occur if both are at the same level, unless the gas in the pipe between them does not move.

The paper is organized as follows: in Sections 2 and 3 the mathematical model is established (further details are given in Appendix A). Then, in Section 4 the existence of a solution to this model is proved as well as uniqueness in the case of flat topography. At the end, two appendices deal, respectively, with obtaining the model equations from the conservation principles of continuum thermomechanics and with some elements on graph theory.

2. Mathematical model: data and unknowns

The goal of this section is to introduce a mathematical model for steady-state gas flow in a gas transportation network which will be subsequently analyzed in Section 4. A gas transportation network consists of different elements and devices such as entry points, exit points, gas pipelines with different sizes, compression stations, flow control valves (FCV), closing valves and pressure control valves (PCV). Its topology can be represented by a directed graph corresponding to the following choices:

- The *nodes* represent the gas supply points, the gas consumption points, the underground storages, the suction or discharge points in a compression station, the interconnection points among pipelines, and the points where the latter change diameter or some other property.
- The *edges* represent the pipelines, the compressors (each compressor links the suction node and the discharge node by the ratio of their increasing pressures), the flow control valves (FCV) (where the mass flow rate is imposed), the closed closing valves (where the mass flow is zero), the bypasses or open closing valves, the pressure control valves (PCV) (which link two nodes by the ratio of their decreasing pressures).