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Modelling and analysis of global resurgence of mumps: A multi-group epidemic model with asymptomatic infection, general vaccinated and exposed distributions

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HIGHLIGHTS

- A novel multi-group epidemic model for studying global resurgence of mumps is formulated.
- Asymptomatic infection, general vaccinated and exposed distributions, and nonlinear incidence are incorporated.
- Global stability of the model proposed is completely solved.
- The threshold dynamics of the ODEs multi-group model with imperfect vaccination is achieved.

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ABSTRACT

Based on the possible mechanism of the recent global resurgence of mumps, a novel multi-group SVEIAR epidemic model with infinite distributed delays of vaccination and latency, asymptomatic infection and nonlinear incidence is formulated, which is also equivalent to the multi-group version with ages of vaccination and latency. Here, it is assumed that the vaccine confers complete protection against infection. By constructing Lyapunov functionals, it is shown that the disease will die out if the vaccinated reproduction number $\mathfrak{R}_v \leq 1$ and the disease becomes endemic if $\mathfrak{R}_v > 1$. Moreover, the threshold dynamics of the multi-group SVEIAR epidemic model of ordinary differential equations (ODEs) with imperfect vaccination is also established. Our main conclusions generalize and improve some existing results. In the end, it is found that the possible reasons for occurrence of backward bifurcation in epidemic models with vaccination may include partial immunity, standard incidence by comparing our results obtained with the existing literatures.

1. Introduction

As one of the oldest recognized diseases, mumps is an acute infection caused by the mumps virus (MuV), a member of the RNA viruses in the family Paramyxoviridae family, which is spread by salivary or

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respiratory secretions, and close contact with infected individuals [1]. Characterized by painful swelling of the parotid glands, mumps can progress to encephalitis, meningitis, orchitis, pancreatitis, nephritis and deafness. Following the introduction of the routine mumps vaccination programmes, the disease was once on the verge of elimination with high two-dose coverage of the measles–mumps–rubella (MMR) vaccines in some countries, such as USA [1], Ireland [2], England and Wales [3]. However, mumps has been experiencing a global resurgence among adolescents and young adults at present [1–3].

Thereinto, Japan has one of the highest rates of mumps in developed countries due to complete cessation of mumps vaccination [4]. In fact, the increase in mumps cases is not restricted to non-vaccinated individuals, on the contrary, outbreaks of mumps do not only involve a high percentage of persons with a vaccination history [5,6] but also are on the rise in the vaccinated communities all over the world [7]. It was found that waning vaccine-induced immunity (secondary vaccine failure) becomes one of the main reasons for the resurgence [5,8–10] despite high protection (about 95% [11]) against mumps. Meanwhile, infection with MuV is asymptomatic/inapparent (which contrasts with symptomatic/apparent infection) in one-third of cases [1,6,12,13], and the patients and the asymptomatic individuals are the main source of infection. The latter with potential contagion (seeing [3,6,12]) has also contributed a great deal to the recent revival of mumps. The heterogeneity of populations may play the vital role in mumps transmission, such as age [2,3], and geographic distributions [10]. As mentioned before, the mumps cases reported during the resurgence are mainly involved in children and young adults in the age of 4–24 years, especially in the age group 15–24 years [2,3], even though widespread population at all ages is vulnerable to infection. Eriksen et al. [10] observed the varied mumps incidences in 18 European countries with different mumps vaccination programmes.

In mathematical epidemiology, multi-group epidemic models have been developed to capture heterogeneity of the disease transmission by modelling separately within-group and inter-group interactions, e.g., [14], which reflects that diverse or dispersed populations with different gender, age, education levels, etc., may admit varied contact patterns, modes of transmission, or spatial distributions. Based on a graph-theoretic approach developed by Guo et al. [15,16], the uniqueness and global stability of endemic equilibrium for multi-group epidemic models have been effectively resolved [17–19]. Recently, Ding and Ding [19] achieved global stability of a multi-group epidemic model with vaccination and delays. Wang et al. [20] established the threshold dynamics of a multi-group susceptible–vaccinated–exposed–infectious–recovered (SVEIR) with distributed delay describing varying infectivity, where it was assumed that susceptible individuals who obtain immunity during or after vaccination enter into vaccinated class and then move into recovered class eventually. Many epidemic models with age of vaccination have been proposed to reflect the waning of vaccine-induced immunity with time since vaccination [21–25]. Equivalently, several epidemic models with a general vaccinated distribution [26,27] can embody the immunity wanes since vaccination. Arino et al. [26] proposed an SIVS model with imperfect vaccine and general waning function, and observed the existence of a backward bifurcation for the case where the vaccine-induced immunity wanes exponentially or the waning time of immunity is a constant. Li and Ma [27] studied the dynamics of an SIVS epidemic model with perfect vaccine and two special probability distributions of vaccination. However, there have been few works completely resolving the global stability of epidemic models with a general vaccinated distribution. The major purpose of this contribution is to formulate a compartmental model that consists in exploring the impacts of general vaccinated distribution, asymptomatic infection and heterogeneity of populations on mumps transmission, and establish its global stability to improve the existing results.

Now, we first consider within-group interactions in certain group. Suppose that new recruits (including travel, immigration, etc.) entering susceptible class $S(t)$ at time t are given by a constant rate Λ without vaccination. A fraction $p \in [0, 1)$ of the new recruits vaccinate the available vaccine against mumps and enter vaccinated class $V(t)$ (usually, we take no account of mumps vaccination for newborns [2]). Meanwhile, susceptible class also get vaccinated at rate v. Note that mumps vaccine can produce a clinical protection against mumps in about 95% of recipients. For the sake of simplicity, we consider certain vaccine offers