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Traveling waves for a diffusive SEIR epidemic model with non-local reaction and with standard incidences

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ABSTRACT

This paper is devoted to the existence of the traveling waves of the equations describing a diffusive SEIR model with non-local reaction between the infected and the susceptible. The existence of traveling waves depends on the minimal speed c^* and basic reproduction rate β/γ. We use the Laplace transform and the Schauder fixed point theorem to get the existence and non-existence of traveling waves in our paper. We also give some numerical results of the minimal wave speed.

1. Introduction

In 1927, Kermack and McKendrick [1] proposed the Kermack–McKendrick equations

$$\frac{d}{dt}S(t) = -\beta S(t)I(t),$$
$$\frac{d}{dt}I(t) = \beta S(t)I(t) - \gamma I(t),$$
$$\frac{d}{dt}R(t) = \gamma I(t)$$

to describe the SIR epidemic model, where S is the number of susceptible population, while I and R denote the number of the infectious and the recovered, respectively. β is the transmission rate and γ is the removing rate of I.

If the spacial diffusion is taken into account, the Kermack–McKendrick equations become

$$\frac{\partial S(x,t)}{\partial t} = d_1 \frac{\partial^2 S(x,t)}{\partial x^2} - \beta S(x,t)I(x,t),$$
$$\frac{\partial I(x,t)}{\partial t} = d_2 \frac{\partial^2 I(x,t)}{\partial x^2} + \beta S(x,t)I(x,t) - \gamma I(x,t),$$

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$$\frac{\partial R(x,t)}{\partial t} = d_3 \frac{\partial^2 R(x,t)}{\partial x^2} + \gamma I(x,t).$$

If we consider the above model with the standard incidences, we have

$$\begin{aligned}
\frac{\partial S(x,t)}{\partial t} &= d_1 \frac{\partial^2 S(x,t)}{\partial x^2} - \frac{\beta S(x,t) I(x,t)}{S(x,t)+I(x,t)},\\
\frac{\partial I(x,t)}{\partial t} &= d_2 \frac{\partial^2 I(x,t)}{\partial x^2} + \frac{\beta S(x,t) I(x,t)}{S(x,t)+I(x,t)} - \gamma I(x,t),\\
\frac{\partial R(x,t)}{\partial t} &= d_3 \frac{\partial^2 R(x,t)}{\partial x^2} + \gamma I(x,t),
\end{aligned}$$

where d_1, d_2, d_3 are the rate of diffusion. There is no R in the first two equalities of the above system by the assumption that the recovered are removed from population. Brauer gave the detailed epidemiological explanation of this assumption in [2].

Many works are done on the traveling-wave solutions of the above system [3]. Wang et al. [4] proved the existence of traveling wave solution of this system by using Schauder fixed point theorem. For $R_0 := \beta/\gamma > 1$ and $c > c^* := 2\sqrt{d_2(\beta-\gamma)}$, the above system has a traveling wave solution $(S(x+ct), I(x+ct))$ satisfying the boundary conditions $S(\infty) = S_\infty$, $S(-\infty) = S_{-\infty}$, $I(\pm\infty) = 0$ and $S_{-\infty} > S_\infty$. Furthermore, there exists no non-negative non-trivial traveling wave solution if $0 < c < c^*$ or $R_0 \le 1$. More works have been done to get the traveling-wave solutions considering the diffusion term is non-local in space or with distributed time delay (see [5–12]).

The corresponding SEIR model is

$$\begin{aligned}
\frac{\partial S(x,t)}{\partial t} &= d_1 \frac{\partial^2 S(x,t)}{\partial x^2} - \frac{\beta S(x,t) I(x,t)}{S(x,t)+E(x,t)+I(x,t)},\\
\frac{\partial E(x,t)}{\partial t} &= d_2 \frac{\partial^2 E(x,t)}{\partial x^2} + \frac{\beta S(x,t) I(x,t)}{S(x,t)+E(x,t)+I(x,t)} - \alpha E(x,t),\\
\frac{\partial I(x,t)}{\partial t} &= d_3 \frac{\partial^2 I(x,t)}{\partial x^2} + \alpha E(x,t) - \gamma I(x,t),\\
\frac{\partial R(x,t)}{\partial t} &= d_4 \frac{\partial^2 R(x,t)}{\partial x^2} + \gamma I(x,t),
\end{aligned}$$

where E is the density of the exposed individuals and α is the rate of the exposed turn to infected. Obviously, $1/\alpha$ is the average period of exposed population become infected. We use the assumption that the exposed individuals are of no infectiousness and their diffusive rate is d_2 is not the same as d_3, which is the diffusive rate of the infected.

In this paper we consider an SEIR model with non-local reaction as follows

$$\frac{\partial S(x,t)}{\partial t} = d_1 \frac{\partial^2 S(x,t)}{\partial x^2} - \frac{\beta S(x,t) \int_{-\infty}^{\infty} I(x-y,t)K(y)dy}{S(x,t)+E(x,t)+\int_{-\infty}^{\infty} I(x-y,t)K(y)dy}, \tag{1.1}$$

$$\frac{\partial E(x,t)}{\partial t} = d_2 \frac{\partial^2 E(x,t)}{\partial x^2} + \frac{\beta S(x,t) \int_{-\infty}^{\infty} I(x-y,t)K(y)dy}{S(x,t)+E(x,t)+\int_{-\infty}^{\infty} I(x-y,t)K(y)dy} - \alpha E(x,t), \tag{1.2}$$

$$\frac{\partial I(x,t)}{\partial t} = d_3 \frac{\partial^2 I(x,t)}{\partial x^2} + \alpha E(x,t) - \gamma I(x,t), \tag{1.3}$$

$$\frac{\partial R(x,t)}{\partial t} = d_4 \frac{\partial^2 R(x,t)}{\partial x^2} + \gamma I(x,t), \tag{1.4}$$

where the reaction kernel $K(x)$ satisfies the following assumptions: