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On the lack of exponential stability for an elastic–viscoelastic waves interaction system[☆]

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ABSTRACT

In this paper, we consider an interaction system in which a wave and a viscoelastic wave equation evolve in two bounded domains, with natural transmission conditions at a common interface. We show the lack of uniform decay of solutions in general domains. The method is based on the construction of ray-like solutions by means of geometric optics expansions and a careful analysis of the transfer of the energy at the interface.

1. Introduction

Engineering applications give rise to fluid–structure interactions, composite laminates in smart materials and structures, structural-acoustic systems, and other interactive physical process, which are modeled by coupled partial differential equations (PDEs). Control design and stability analysis for such systems have become active over the past decades, see [1–22] and references therein. The stability analysis for heat-wave interaction systems were treated in [2,10,19,22]. Later, the uniform stabilization, polynomial stability and backward uniqueness problems for the fluid–structure interaction system were analyzed in [3–7,9,14]. More recently, [1,20] studies stabilization problems of wave-plate and heat-plate interaction systems, respectively.

In this paper, we study the exponential stability property of an interaction system. This system consists of a wave and a viscoelastic wave equation coupled through an interface with suitable transmission conditions. More precisely, let $\Omega \subset \mathbb{R}^n$ ($n \in \mathbb{N}$) be a bounded domain with C^2 boundary Γ, Ω_1 be a sub-domain of Ω and $\Omega_2 = \Omega \backslash \overline{\Omega}_1$. Denote by γ the interface, $\Gamma_i = \partial\Omega_i \backslash \gamma$ ($i = 1, 2$), and ν_i the unit outward normal vector

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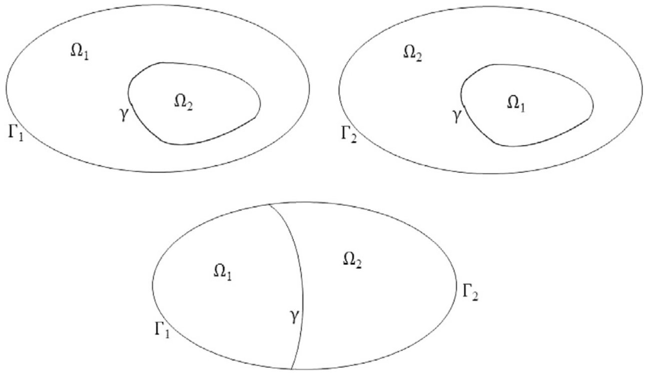

Fig. 1. Geometric description of the elastic–viscoelastic waves interaction model.

of $\partial\Omega_i$ $(i = 1, 2)$. Assume that γ is a nonempty open subset of $\partial\Omega_j$. The PDEs of the interaction system are as follows.

$$\begin{cases} \partial_{tt}y(t,x) - \Delta y(t,x) - \Delta\partial_t y(t,x) = 0 & \text{in } (0,\infty)\times\Omega_1, \\ \partial_{tt}z(t,x) - \Delta z(t,x) = 0 & \text{in } (0,\infty)\times\Omega_2, \\ y(t,x) = 0 & \text{on } (0,\infty)\times\Gamma_1, \\ z(t,x) = 0 & \text{on } (0,\infty)\times\Gamma_2, \\ y(t,x) = z(t,x) & \text{on } (0,\infty)\times\gamma, \\ \partial_{\nu_1}y(t,x) + \partial_{\nu_1}\partial_t y(t,x) = -\partial_{\nu_2}z(t,x) & \text{on } (0,\infty)\times\gamma, \\ y(0,x) = y^0, \qquad \partial_t y(0,x) = y^1 & \text{in } \Omega_1, \\ z(0,x) = z^0, \qquad \partial_t z(0,x) = z^1 & \text{in } \Omega_2, \end{cases} \tag{1.1}$$

where the variable y denotes the wave component with viscoelastic damping, and z, the pure wave solution (see Fig. 1).

It is clear that the dissipative mechanism acting on system (1.1) is viscoelastic damping $\Delta\partial_t y$, which is only effective on Ω_1. Actually, the energy of system (1.1) is defined by

$$E(t) \doteq E\big(y,z\big)(t) \doteq \frac{1}{2}\Big[\int_{\Omega_1}\big(|\nabla y(t)|^2 + |y_t(t)|^2\big)dx + \int_{\Omega_2}\big(|\nabla z(t)|^2 + |z_t(t)|^2\big)dx\Big]. \tag{1.2}$$

It is easy to check that

$$\frac{d}{dt}E(t) = -\int_{\Omega_1}|\nabla\partial_t y(t,x)|^2 dx. \tag{1.3}$$

Therefore, the energy of (1.1) is decreasing with respect to time. Naturally, one hopes to know whether the dissipation is strong enough to produce the exponential stability of the energy.

Note that system (1.1) could also be viewed as a special case of the following model.

$$\begin{cases} \partial_{tt}Z - \text{div}\,[\nabla Z + D(x)\nabla\partial_t Z] = 0 & \text{in } (0,\infty)\times\Omega, \\ Z = 0 & \text{on } (0,\infty)\times\Gamma, \\ Z(0) = Z_0 & \text{in } \Omega, \end{cases} \tag{1.4}$$

with

$$Z = \begin{cases} y, & x\in\Omega_1, \\ z, & x\in\Omega_2; \end{cases} \qquad D(x) = \begin{cases} d(x)\ge 0, & x\in\Omega_1, \\ 0, & x\in\Omega_2. \end{cases}$$

The well-posedness and strong stability of system (1.4) was proven in [16] when $D \in L^\infty(\Omega)$. If the material parameter D is smooth enough at the interface, say $D(x) \in C^2(\overline{\Omega})$, the energy of system (1.4) decays