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Homogenization of parabolic problem with nonlinear transmission condition

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ABSTRACT

In this paper, we investigate the periodic homogenization of nonlinear parabolic equation arising from heat exchange in composite material problems. This problem, defined in periodical domain, is nonlinear at the interface. This nonlinearity models the heat radiation on the interface, which constitutes the transmission boundary conditions, between the two components of the material. The main challenge is, first, to show the well-posedness of the microscopic problem using the topological degree of Leray–Schauder tools. Then, we apply the two scale convergence to identify the equivalent macroscopic model using homogenization techniques. Finally, in order to confirm the efficiency of the homogenization process, we present some numerical results obtained via finite element approximation.

1. Introduction

The composite material is a macroscopic combination of two or more different materials, separated by a recognizable interface. The use of composites is not restricted only to structural properties, but can also be used in some issues related to electrical, thermal, tribological, and environmental applications. One of the important challenges is to have material which achieves a particular balance of properties for a given range of applications [1]. The resulting desired composite material is well structured compared with two or more given constituents material. However, in heat exchange in composite material problems, even if it have great structural properties, it is difficult to determine the effective thermal conductivity tensor associated to thermal behavior. This is due to the temperature evolution during the injection process, of which the

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control of the optimality of the obtained piece requires some knowledge and control of thermal cycle. A natural way to overcome those difficulties is to replace the composite with a kind of equivalent material model. This procedure is usually called periodic homogenization [1]. In many industrial areas, the multi-scale nature of the problem is imposed by the microstructure of the material under consideration. These multi-scale problems lead to complex geometric domain with unrealizable details, describing a large scale model. Moreover, the presence of the flow during the injection process in the material requires rigorous numerical simulation of temperature evolution. The relevant question is: How to obtain a model with less complicated geometric domain, conserving the behavior of the local microscopic temperature of each material constituents?

Numerous approaches were proposed to treat the challenges deriving from the behavior of the composite material. The first class aims to carry out experimental measurements with one or several devices to get all the components of the tensor [2]. The second one intends to use predictive models based on homogenization theory from the properties and the arrangement at the microscopic scale. It consists to find a global systematic model that replaces the composite model and conserving, at the same time, the microscopic behavior of the material. The first studies done within this framework of periodic homogenization and leading to rigorous macroscopic models can be found for example in [3,4]. This approach leads, in mathematical point of view, to good approximations of both the heat flux and temperature in the interior zone of the structure with simple geometry domain.

In this paper, we consider a homogenization problem of fluid flow in porous media governed by a parabolic problem with a nonlinear boundary transmission conditions and we suppose that the composite piece is made of a periodically-distributed microstructure. At this stage, we note that there are many previous contributions on the homogenization of fluid flow in porous media see for example [3,5] and references therein. In particular, the elliptic problem in linear and nonlinear cases associated to different geometries or scalings has been extensively studied (see for example [6–8]). While the studies of the homogenization of nonlinear parabolic problems with bounded or Lipschitz nonlinear boundary conditions are investigated in [9,10]. In this context, we consider here a macroscopic model resulting from the evolution of the temperature in the composite material. It is governed by parabolic problem with a more general nonlinear term of heat exchange in the boundary transmission conditions, between the two components. Our main goal, in the homogenization process, is to show the well-posedness of this problem and prove the convergence of a sequence of solutions of microscopic problems to the solution of the macroscopic one, in an appropriate functional space, using less regularity assumptions on the conductivity, the velocity and the initial data, than those used in the previous work [9,11,10,12]. In fact, although the homogenization process is standard, this study still has some difficulties in our situation lying in the fact that the problem is strongly coupled (fluid/solid) and that the considered heat exchange term in the Neumann boundary condition is strongly nonlinear. To overcome these difficulties, we show first the existence result by opting for the topological degree of Leray–Schauder tools, which is more powerful and more general and often easier to use than the classical fixed point theorems [13]. We note that in this case, a priori estimation of the solution and the compactness of the mapping under consideration need a special attention. Then we show the uniqueness of the fixed point. The second main result is the upscaling of our problem by periodic homogenization, more precisely, it is devoted to prove the convergence theorem using the two-scale convergence [3].

This paper is organized as follows: first, we introduce the microscopic geometry, the model equations, data assumptions and basic estimates. In Section 2, we give the weak formulation of the model and provide a priori estimates, which allow us to show the solvability of the microscopic problem and the convergence of the microscopic solution to the macroscopic one. The existence and uniqueness of the model by the topological degree of Leray–Schauder are investigated in Section 3. Then, in Section 4, we show the convergence result and identify the macroscopic model, based on two-scale convergence. In the last section, we give some numerical results, showing the efficiency of theoretical results.