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Global existence and well-posedness for the FENE dumbbell model of polymeric flows

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ABSTRACT

In this paper we mainly investigate the Cauchy problem of the finite extensible nonlinear elastic (FENE) dumbbell model with dimension $d \geq 2$. We first proved the local well-posedness for the FENE model in Besov spaces by using the Littlewood–Paley theory. Then by an accurate estimate we get a blow-up criterion. Moreover, if the initial data is a small perturbation around equilibrium, we obtain a global existence result. Our obtained results generalize and cover recent results in Masmoudi (2008).

1. Introduction

In this paper we consider the finite extensible nonlinear elastic (FENE) dumbbell model [1]:

$$\begin{cases} \partial_t u + (u \cdot \nabla)u - \nu \Delta u + \nabla P = div\tau, \quad divu = 0, \\ \partial_t \psi + (u \cdot \nabla)\psi = div_R[-\nabla u \cdot R\psi + \beta \nabla_R \psi + \nabla_R \mathcal{U}\psi], \\ \tau_{ij} = \int_B (R_i \nabla_j \mathcal{U})\psi dR, \\ u|_{t=0} = u_0, \qquad \psi|_{t=0} = \psi_0, \\ (\beta \nabla_R \psi + \nabla_R \mathcal{U}\psi) \cdot n = 0 \quad \text{on } \partial B(0, R_0). \end{cases} \tag{1.1}$$

In (1.1) $\psi(t, x, R)$ denotes the distribution function for the internal configuration and $u(t, x)$ stands for the velocity of the polymeric liquid, where $x \in \mathbb{R}^d$ and $d \geq 2$ means the dimension. Here the polymer elongation R is bounded in ball $B = B(0, R_0)$ of $\mathbb{R}^d$ which means that the extensibility of the polymers is finite. $\beta = \frac{2k_B T_a}{\lambda}$,

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where k_B is the Boltzmann constant, T_a is the absolute temperature and λ is the friction coefficient. $\nu > 0$ is the viscosity of the fluid, τ is an additional stress tensor and P is the pressure. The Reynolds number $Re = \frac{\gamma}{\nu}$ with $\gamma \in (0,1)$ and the density $\rho = \int_B \psi dR$. Moreover the potential $\mathcal{U}(R) = -k\log(1-(\frac{|R|}{|R_0|})^2)$ for some constant $k > 0$.

This model describes the system coupling fluids and polymers. The system is of great interest in many branches of physics, chemistry, and biology, see [1,2]. In this model, a polymer is idealized as an "elastic dumbbell" consisting of two "beads" joined by a spring that can be modeled by a vector R. At the level of liquid, the system couples the Navier–Stokes equation for the fluid velocity with a Fokker–Planck equation describing the evolution of the polymer density. This is a micro–macro model (for more details, one can refer to [1–3]).

In the paper we will take $\beta = 1$ and $R_0 = 1$. Notice that (u,ψ) with $u = 0$ and

$$\psi_\infty(R) = \frac{e^{-\mathcal{U}(R)}}{\int_B e^{-\mathcal{U}(R)}dR} = \frac{(1-|R|^2)^k}{\int_B (1-|R|^2)^k dR},$$

is a stationary solution of (1.1). Thus we can rewrite (1.1) for the following system:

$$\begin{cases} \partial_t u + (u\cdot\nabla)u - \nu\Delta u + \nabla P = div\tau, \quad divu = 0, \\ \partial_t\psi + (u\cdot\nabla)\psi = div_R\left[-\nabla u\cdot R\psi + \psi_\infty\nabla_R\frac{\psi}{\psi_\infty}\right], \\ \tau_{ij} = \int_B (R_i\nabla_j\mathcal{U})\psi dR, \\ u|_{t=0} = u_0, \qquad \psi|_{t=0} = \psi_0, \\ \psi_\infty\nabla_R\frac{\psi}{\psi_\infty}\cdot n = 0 \quad \text{on } \partial B(0,1). \end{cases} \tag{1.2}$$

We have to add a boundary condition to insure the conservation of ψ, namely, $(-\nabla u\cdot R\psi + \psi_\infty\nabla_R\frac{\psi}{\psi_\infty})\cdot n = 0$. The second equation in (1.2) can be understood in the weak sense: for any function $g(R) \in C^1(B)$, we have

$$\partial_t\int_B g\psi dR + (u\cdot\nabla)\int_B g\psi dR = -\int_B \nabla_R g\left[-\nabla u\cdot R\psi + \psi_\infty\nabla_R\frac{\psi}{\psi_\infty}\right]dR.$$

Definition 1.1. Assume that $u_0 \in D'(\mathbb{R}^d)$ and $\psi_0 \in D'(\mathbb{R}^d\times B)$. A couple of functions $(u,\psi) \in C([0,T]; D'(\mathbb{R}^d)) \times C([0,T]; D'(\mathbb{R}^d\times B))$ with $div\ u = 0$ is called a solution for (1.2) if $u\otimes u$, P, $\tau \in L^1((0,T); D'(\mathbb{R}^d)), u\psi$, $\nabla u\cdot R\psi$, $\psi_\infty\nabla_R\frac{\psi}{\psi_\infty} \in L^1((0,T); D'(\mathbb{R}^d\times B))$, and for each $(v,\phi) \in C^1([0,T]; C_0^\infty(\mathbb{R}^d)) \times C^1([0,T]; C_0^\infty(\mathbb{R}^d\times B))$ with $v(T) = 0$ and $\phi(T) = 0$, we have

$$\int_0^T\int_{\mathbb{R}^d} u\partial_t v + (u\otimes u):\nabla v - \nu u\Delta v + P\cdot div\ v = \int_0^T\int_{\mathbb{R}^d}\tau : v + \int_{\mathbb{R}^d} u_0 v_0, \tag{1.3}$$

$$\int_0^T\int_{\mathbb{R}^d\times B}\psi\partial_t\phi + u\psi\cdot\nabla_x\phi = \int_0^T\int_{\mathbb{R}^d\times B}\left[-\nabla u\cdot R\psi + \psi_\infty\nabla_R\frac{\psi}{\psi_\infty}\right]\cdot\nabla_R\phi + \int_{\mathbb{R}^d\times B}\psi_0\phi_0. \tag{1.4}$$

Let us mention that the earliest local well-posedness for (1.1) was established by Renardy in [4], where the author considered the Dirichlet problem with $d = 3$ for smooth boundary and proved local existence for (1.1) in $\bigcap_{i=0}^4 C^i([0,T]; H^{4-i}) \times \bigcap_{i=0}^3\bigcap_{j=0}^{3-i} C^i([0,T]; H^j)$ with potential $\mathcal{U}(R) = (1-|R|^2)^{1-\sigma}$ for $\sigma > 1$. Later, Jourdain, Lelièvre, and Le Bris [5] proved local existence of a stochastic differential equation with potential $\mathcal{U}(R) = -k\log(1-|R|^2)$ in the case $k > 3$ for a Couette flow. Zhang and Zhang [6] proved local well-posedness of (1.1) with $d = 3$ in $\bigcap_{i=0}^2 H^i([0,T]; H^{4-2i}) \times \bigcap_{i=0}^1 H^i([0,T]; H^{3-2i})$ for $k > 38$. Lin, Zhang, and Zhang [7] proved global well-posedness of (1.2) with $d = 2$ for $k > 6$ in $C([0,T]; H^s) \times C([0,T]; H^s(R^2; H_0^1(D)))$, where $s \geq 3$. Masmoudi [2] proved local well-posedness of (1.2) in $C([0,T); H^s) \times C([0,T); H^s(R^d; L^2))$, $s > 1 + \frac{d}{2}$ and global well-posedness of (1.2) when the initial data is perturbation around equilibrium for $k > 0$. In