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Corrigendum

Corrigendum to "Renormalised solutions in thermo-visco-plasticity for a Norton–Hoff type model. Part I: The truncated case" [Nonlinear Anal. RWA 28 (2016) 140–152]

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ABSTRACT

We correct an error in the original paper [Renormalised solutions in thermo-visco-plasticity for a Norton–Hoff type model. Part I: The truncated case, Nonlinear Anal. RWA 28 (2016) 140–152]. The proof of Theorem 3.2 is wrong and therefore we present here correct proof based on the same methods.

1. Introduction

During review of the article *Renormalised solutions in thermo-visco-plasticity for a Norton–Hoff type model. Part II: the limit case, Nonlinear Analysis-Real World Applications, 31 (2016), p. 643-660* one of the reviewers has read the first part entitled *Renormalised solutions in thermo-visco-plasticity for a Norton–Hoff type model. Part I: the truncated case, Nonlinear Analysis-Real World Applications, 28 (2016), p. 140-152.* He/She has observed that the proof of Theorem 3.2 in the first part is not completely justified. After this remark, we have noticed that the proof of Theorem 3.2 is unclear while the statement of Theorem 3.2 is correct (without uniqueness). In this corrigendum we present the right statement of Theorem 3.2 and its proof. The main idea of the proof is also based on a fixed point argument.

2. Theorem 3.2

Suppose that the given data satisfy all requirements of Theorem 1.2. *For all* $\lambda > 0$ *the system* (3.1) *with initial–boundary conditions* (3.2) *and* (3.3) *possesses global in time solution* $(u, \varepsilon^p, \theta)$ *such that*

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$$(u, \varepsilon^p) \in H^1(0, \mathfrak{T}; H^1(\Omega; \mathbb{R}^3)) \times W^{1,\infty}(0, \mathfrak{T}; L^2(\Omega; \mathcal{S}^3_{\mathrm{dev}})),$$
$$\theta \in L^\infty(0, \mathfrak{T}; H^1(\Omega; \mathbb{R})) \text{ and } \theta_t \in L^2(0, \mathfrak{T}; L^2(\Omega; \mathbb{R})).$$

Proof: We are going to use the Schaefer's fixed point theorem. Fix $\theta^\star \in L^r(0, \mathfrak{T}; W^{1,r}(\Omega; \mathbb{R}))$, where $r \in (1, 2)$. Let us consider the following initial–boundary value problem

$$\begin{aligned} \theta_t - \Delta\theta &= \mathcal{T}_{\frac{1}{\epsilon}}\Big(\varepsilon^p_t \cdot T\Big) - f\big(\mathcal{T}_{\frac{1}{\epsilon}}(\theta^\star + \tilde{\theta})\big)\operatorname{div} u_t\,, \\ \frac{\partial\theta}{\partial n}_{|\partial\Omega\times(0,\mathfrak{T})} &= 0, \qquad \theta(0) = \theta_0, \end{aligned} \tag{1}$$

where the functions u, ε^p, T are the unique solution of the problem (3.4) (in the published article) with the input function $\theta^\star$. The right-hand side of (1) belongs to $L^2(0, \mathfrak{T}; L^2(\Omega; \mathbb{R}))$. Using the maximal regularity (see Amann [1] and [2]) of solutions to linear parabolic problems (1) we obtain that the problem (1) admits a solution in the class

$$\theta \in L^r(0, \mathfrak{T}; W^{2,r}(\Omega; \mathbb{R}))\,, \qquad \theta_t \in L^r(0, \mathfrak{T}; L^r(\Omega; \mathbb{R}))\,.$$

Hence, we have defined an operator

$$L^r(0, \mathfrak{T}; W^{1,r}(\Omega; \mathbb{R})) \ni \theta^\star \to \mathcal{R}(\theta^\star) = \theta \in L^r(0, \mathfrak{T}; W^{1,r}(\Omega; \mathbb{R})).$$

Next we divide the proof into three steps.

Step 1 (continuity of the operator $\mathcal{R}$)

Let us assume that $\theta^\star_n \to \theta^\star$ in $L^r(0, \mathfrak{T}; W^{1,r}(\Omega; \mathbb{R}))$. Lemma 3.1 (in the published article) yields that for all $n = 1, 2, \ldots$, there exists a global in time solution

$$(u_n, T_n, \varepsilon^p_n) \in H^1(0, \mathfrak{T}; H^1(\Omega; \mathbb{R}^3)) \times H^1(0, \mathfrak{T}; L^2(\Omega; \mathcal{S}^3)) \times W^{1,\infty}(0, \mathfrak{T}; L^2(\Omega; \mathcal{S}^3_{\mathrm{dev}}))$$

and a global in time solution

$$(u, T, \varepsilon^p) \in H^1(0, \mathfrak{T}; H^1(\Omega; \mathbb{R}^3)) \times H^1(0, \mathfrak{T}; L^2(\Omega; \mathcal{S}^3)) \times W^{1,\infty}(0, \mathfrak{T}; L^2(\Omega; \mathcal{S}^3_{\mathrm{dev}}))$$

of the system (3.4) with the input functions $\theta^\star_n$ and $\theta^\star$, respectively. Moreover the weak formulation of the system (3.4) yields

$$\begin{aligned} &\int_\Omega \big\langle T_n - T, \varepsilon(v)\big\rangle\,\mathrm{d}x + \int_\Omega \big\langle \mathbb{C}\big(\varepsilon((u_n)_t) - \varepsilon(u_t)\big), \varepsilon(v)\big\rangle\,\mathrm{d}x \\ &\quad = \int_\Omega \Big(f\big(\mathcal{T}_{\frac{1}{\epsilon}}(\theta^\star_n + \tilde{\theta})\big) - f\big(\mathcal{T}_{\frac{1}{\epsilon}}(\theta^\star + \tilde{\theta})\big)\Big)\operatorname{div} v\,\mathrm{d}x \end{aligned} \tag{2}$$

for all $v \in H^1_0(\Omega, \mathbb{R}^3)$. Putting $v = (u_n)_t - u_t$ in (2) we obtain

$$\begin{aligned} &\frac{1}{2}\frac{d}{dt}\big(\int_\Omega \big\langle \mathbb{C}^{-1}(T_n - T), T_n - T\big\rangle\,\mathrm{d}x\big) + \int_\Omega \big\langle \mathbb{C}\big(\varepsilon((u_n)_t) - \varepsilon(u_t)\big), \varepsilon((u_n)_t) - \varepsilon(u_t)\big\rangle\,\mathrm{d}x \\ &\quad = -\int_\Omega \big\langle T_n - T, (\varepsilon^p_n)_t - \varepsilon^p_t\big\rangle\,\mathrm{d}x \\ &\quad + \int_\Omega \big(f\big(\mathcal{T}_{\frac{1}{\epsilon}}(\theta^\star_n + \tilde{\theta})\big) - f\big(\mathcal{T}_{\frac{1}{\epsilon}}(\theta^\star + \tilde{\theta})\big)\big)\operatorname{div}\big((u_n)_t - u_t\big)\,\mathrm{d}x. \end{aligned} \tag{3}$$

The first integral on the right-hand side of (3) is non positive. Using the Cauchy–Schwarz inequality with small weight and integrating with respect to time in (3) we get

$$\begin{aligned} &\int_\Omega \big\langle \mathbb{C}^{-1}(T_n - T), T_n - T\big\rangle\,\mathrm{d}x + \int_0^t\int_\Omega \big\langle \mathbb{C}\big(\varepsilon((u_n)_t) - \varepsilon(u_t)\big), \varepsilon((u_n)_t) - \varepsilon(u_t)\big\rangle\,\mathrm{d}x\,\mathrm{d}\tau \\ &\quad \le C(\nu)\int_0^t \|f\big(\mathcal{T}_{\frac{1}{\epsilon}}(\theta^\star_n + \tilde{\theta})\big) - f\big(\mathcal{T}_{\frac{1}{\epsilon}}(\theta^\star + \tilde{\theta})\big)\|^2_{L^2(\Omega;\mathbb{R})}\,\mathrm{d}\tau \\ &\quad + \nu\int_0^t \|\varepsilon((u_n)_t) - \varepsilon(u_t)\|^2_{L^2(\Omega;\mathbb{R})}\,\mathrm{d}\tau. \end{aligned} \tag{4}$$