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Classical global solutions for compressible radiative flow in a slab under semi-reflexive boundary conditions

F.S. de Azevedo, E. Sauter*, M. Thompson

Programa de Pós-Graduação em Matemática Aplicada, Universidade Federal do Rio Grande do Sul, Bento Gonçalves, P.O. Box 15080, Porto Alegre - RS, Brazil

HIGHLIGHTS

- Existence was established for radiative–conductive problem coupled with motion.
- The theory of existence is based on a homotopic Leray–Schauder fixed point Theorem.
- The theory of existence was established in classical functional spaces.

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ABSTRACT

Classical global solutions are established for a model of compressible radiative flow in a slab under semi-reflexive boundary conditions using energy–entropy estimates and a homotopic version of the Leray–Schauder fixed point Theorem together with classical Friedman–Schauder estimates for linear second order parabolic equations in boundary Hölder spaces.

1. Introduction

Interest in incorporating radiation effects in compressible fluid dynamics has led to the study of simplified one-dimensional models as, for example, in the papers by Qin and Hu [1], Wang and Xie [2], Zhang and Xie [3] and more recently the studies by Ducomet and Nečasová and Ducomet and Feireisl [4–7], all of which have in mind applications in astrophysics. These one-dimensional models work under the hypotheses of homogeneous

* Corresponding author.
E-mail addresses: fabio.azevedo@ufrgs.br (F.S. de Azevedo), esequia.sauter@ufrgs.br (E. Sauter), mark.thompson@ufrgs.br (M. Thompson).

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Dirichlet–Neumann conditions in the Navier–Stokes–Fourier equations and transparent conditions in the radiative transfer equation, when in fact, this is used. In most of this work the radiative aspects (at the high temperatures appropriate to astrophysical applications) are taken into account introducing both non-linear temperature dependent time and diffusion coefficients in the energy balance equation. This is the case in what the authors of [4] call the infra-relativistic case. The paper [5] works in the context of special relativity theory (see Mihalas and Weibel-Mihalas [8]) but treating matter as a Newtonian fluid with viscous contribution (in this area, fraught with profound conceptual problems, one might refer to the discussion in [9, chapter 9]). The paper [5] uses radiative entropy expressions in terms of the radiative intensity function (see [10,11]) as well as the technique of relative free energy estimates. In [12], Frank et al. study a 3-dimensional model arising from the necessity in modern gas combustion chamber engineering to predict accurately the load of the walls due to thermal radiation. However, the velocity field seems to be treated as a functional parameter and the variation of density ignored. A good part of the emphasis is on the development of entropy–moment approximations for the coupled linear radiative transfer equation to a nonlinear temperature equation (a lucid account of such work is given in the paper by Frank et al. [13]). What is crucially relevant in [12] is that the gas is not thermodynamically isolated from its surroundings and semi-reflexive boundary conditions are taken in the radiative transfer equation together with more complex non-linear Robin type boundary conditions on the gas temperature. The model ignores radiative pressure and, as in [4], radiative entropy. Local thermal equilibrium holds so that the temperature of the gas and radiation are taken to be the same. Note that these are specific model hypothesis and are not determined by asymptotic analysis from a more basic model. Such questions are examined in the papers by Buet and Després [14,15] where non-equilibrium models are discussed in the context of Euler equation coupled with radiation. Subsequent papers [16,17] in more elaborate analysis have dealt with low Mach number flows in simplified geometries under simpler boundary conditions but with chemical reactions and an additional asymptotic parameter, the optical thickness.

In the present paper, we specialise the problem of [12] to an effectively one-dimensional case but with a full non-linear coupling to the compressible gas, thereby generalising the previous work in [18,19]. We establish energy and entropy estimates on classical solutions to the model equations which uses results from [18,19] as well as estimates which follow quite closely those of [3] in order to obtain global a priori estimates in the Hölder Category. These estimates are then used as in [4,5] in an application of a homotopic Leray–Schauder fixed point theorem (see [20]) to prove the existence of global solutions in the Hölder Spaces. In [4,5] this result is a technical one preparatory to establishing a more general result on weak global solutions for large data. Such a result is possible here and we might simply cite the remarks made at the end of [4]. This work is organised as follows: in Section 2 we present the model and establish the mathematical background and formulation. In Section 3 we demonstrate some estimates on the radiative flux. Section 4 is one of the principal parts of our work where we present and prove a series of estimates on the energy and the entropy of the flux which allow us to use in the existence theory. Section 5 is concerned with establishing the properties of the homotopy in the classical Hölder boundary spaces. The analysis of the homotopy is greatly complicated by the fact that the first boundary value problem needs to satisfy second order compatibility conditions. Here the analysis is simplified using a trick involving a change in a dependent velocity variable u introduced by Dafermos [21]. The estimates given in Lemma 8 together with an additional compacity argument in Theorem 9 combined with previous work [22,23] on the linear radiative transfer equation yield the principal result on the existence theory enounced in Corollary 10.

2. The model equations of the radiative gas flow

We study a version of a model for radiative compressible gas flow introduced by Frank et al. [12] in a slab under semi-reflexive boundary conditions for the radiative transfer equation. Our formulation is similar to that studied by Zhang and Xie [3] in a problem of thermally radiative magnetohydrodynamics and is