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Analysis of a general dynamic history-dependent variational–hemivariational inequality[☆]

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ABSTRACT

This paper is devoted to the study of a general dynamic variational–hemivariational inequality with history-dependent operators. These operators appear in a convex potential and in a locally Lipschitz superpotential. The existence and uniqueness of a solution to the inequality problem is explored through a result on a class of nonlinear evolutionary abstract inclusions involving a nonmonotone multivalued term described by the Clarke generalized gradient. The result presented in this paper is new and general. It can be applied to study various dynamic contact problems. As an illustrative example, we apply the theory on a dynamic frictional viscoelastic contact problem in which the contact is modeled by a nonmonotone Clarke subdifferential boundary condition and the friction is described by a version of the Coulomb law of dry friction with the friction bound depending on the total slip.

1. Introduction

Inequality problems in Mechanics can be divided into two main categories: that of variational inequalities, which is mainly concerned with convex functions, and that of hemivariational inequalities, which is concerned with nonconvex locally Lipschitz functions. Both variational and hemivariational inequalities are useful in a wide variety of applications, in Mechanics, Engineering, Economics, etc.

The notion of hemivariational inequality was first introduced and studied by P.D. Panagiotopoulos [1] in connection with contact problems with nonmonotone and possibly multivalued constitutive or interface

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laws for deformable bodies. The literature on the mathematical theory and applications of hemivariational inequalities is growing; some comprehensive references are [2–8].

Variational–hemivariational inequalities can be seen as an effective mathematical tool which combines the variational inequalities and the hemivariational inequalities and involves both convex and nonconvex energy functions. Recently, some papers are found studying various types of variational–hemivariational inequalities. For instance, an adhesive unilateral contact problem between a viscoelastic body and a deformable foundation is treated in [9], where a result is presented on unique solvability for a system consisting of a variational–hemivariational inequality and an ordinary differential equation. In [10,11], results are obtained on the existence and uniqueness of solutions for two different types of variational–hemivariational inequalities and applied these abstract results to study a static frictional contact problem and a quasistatic frictionless contact problem, respectively.

The goal of this paper is to study a general evolutionary variational–hemivariational inequality which involves history-dependent operators. The latter are operators which at any time moment depend on the history of the solution up to this time moment. Analysis of various classes of history-dependent quasistatic hemivariational inequalities and variational–hemivariational inequalities and their applications can be found in [9,12–14,6,11,15,16]. An evolutionary history-dependent variational–hemivariational inequality is studied in [17]. Different from [17], in this paper, we consider a general evolutionary variational–hemivariational inequality without a compact operator in the multivalued term and provide a new existence and uniqueness result. We allow now both convex potential and nonconvex superpotentials to depend on history-dependent operators.

Our abstract results find applications in the variational analysis of a variety of dynamic contact problems. To provide an illustrative example we consider a dynamic frictional contact problem for viscoelastic materials with long memory. In this problem the contact is described with a nonmonotone normal damped response condition with a damping coefficient depending on the normal displacement. The friction is modeled by a version of Coulomb's law of dry friction with the friction bound depending on the total slip. The weak formulation in terms of the unknown velocity of the contact problem leads to a variational–hemivariational inequality with three history-dependent operators to which our abstract results apply. Then we mention two different versions of the problem which can be treated by our abstract existence and uniqueness result.

The paper is structured as follows. In Section 2, we review some basic mathematical notation, definitions and results. In Section 3, we provide an existence and uniqueness result for an abstract subdifferential inclusion of first order considered in the framework of an evolution triple of spaces. Then, in Section 4, we show the unique solvability of a general abstract evolutionary history-dependent variational–hemivariational inequality. Finally, in Section 5, we introduce a new viscoelastic frictional contact problem, present its weak formulation, and apply the abstract result obtained in Section 4 to prove the existence of a unique weak solution to the contact problem.

2. Preliminaries

In this section we briefly review basic definitions on single-valued and multivalued operators in Banach spaces and on subdifferentials which are used later. We refer to [18–21] for more material on these topics.

Let $(X, \|\cdot\|_X)$ be a reflexive Banach space. We denote by X^* its topological dual and by $\langle \cdot,\cdot\rangle_{X^*\times X}$ the duality pairing of X and X^*. Given a set $S \subset X$, we define $\|S\|_X = \sup\{\|s\|_X \mid s \in S\}$. A single-valued operator $A\colon X \to X^*$ is said to be hemicontinuous if the function $\lambda \mapsto \langle A(u+\lambda v), w\rangle_{X^*\times X}$ is continuous on $[0,1]$ for all $u, v, w \in X$. The operator A is demicontinuous if for all $w \in X$, the functional $u \mapsto \langle Au, w\rangle_{X^*\times X}$ is continuous, i.e., A is continuous as a mapping from X to $(w^*\text{–}X^*)$. It is monotone if $\langle Au - Av, u - v\rangle_{X^*\times X} \geq 0$ for all $u, v \in X$. It is known that for a monotone operator $A\colon X \to X^*$