Global Robust Stabilization of Non-triangular Time-delay System with Uncertain Dynamics and Disturbances

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***Abstract*—This work studies the robust stabilization problem of non-triangular delayed nonlinear system. The considered system involves the uncertain dynamics, unknown control coefficient and time delay. By employing a novel Lyapunov-Krasovskii functional and using the backstepping method, we raise a robust controller to guarantee the states of the system bounded. An example is given to verify the validity of the strategy.**

I. Introduction

Nonlinear delayed systems exist in many practical systems and have received extensive attention in recent years, see [1-9]. Among this work, the problems such as state feedback stabilization control [1-3,5], tracking control [4] and output feedback control [6], have been studied extensively. Also, there exist many strategies proposed. For instance, the homogeneous domination method [7], the backstepping method [8,9], the dynamic surface control method [10-11] and so on. However, there are still lots of problems which have not been solved for many reasons, e.g., the complicated structure and the uncertainty.

In this work, we investigate the following system

$$\begin{aligned} \dot{\varepsilon} &= f_0\big(t,\varepsilon,x,x(t-d)\big),\\ \dot{x}_i &= a_i x_{i+1} + f_i\big(t,\varepsilon,x,\,x(t-d),\omega_i\big),\ i=1,\ldots,n-1,\\ \dot{x}_n &= a_n u + f_n\big(t,\varepsilon,x,\,x(t-d),\omega_n\big), \end{aligned} \tag{1}$$

where $x(t) = [x_1(t),\ldots,x_n(t)]^\top \in \mathcal{R}^n$, $\varepsilon(t) \in \mathcal{R}^m$ and $u(t) \in \mathcal{R}$ are the measurable state, the system zero dynamics and the control input, respectively; d is the constant time delay; $x(s) = \phi(s)$, $s \in [-d, 0]$ with $\phi(\cdot)$ being a specified continuous function; $1 \le a_i(t) \le a_j(t), 1 \le i \le j \le n$ are unknown functions; $\omega_k(t)$ are the disturbances, $f_k(\cdot), k = 1, \ldots, n$ are continuous functions.

As can be seen, the system here is not easy to stabilize since it includes uncertain dynamics, time-delay, disturbances and unknown coefficients. Moreover, the nonlinear terms of system (1) may be not in triangular form, which also adds the obstacles for control design. In next section, by using the backstepping method and by introducing a new L-K functional, we will consider how to design a robust controller for it.

II. Control of nonlinear time-delay system

A. Preliminaries and problem statement

Firstly, the assumptions are imposed as:

Assumption 1 *The subsystem ε is input-to-state stable, i.e., we can find a positive definite function $U_0(\varepsilon)$ such that*

$$\begin{aligned} c_1\|\varepsilon\|^2 \le U_0(\varepsilon) \le c_2\|\varepsilon\|^2,\\ \dot{U}_0(\varepsilon) \le -l_1\|\varepsilon\|^2 + l_2(|x_1|^2 + |x_1(t-d)|^2), \end{aligned}$$

where c_j and l_j, $j = 1, 2$ are positive constants.

Assumption 2 *For $i = 1, \ldots, n$, there holds*

$$\begin{aligned} |M^{-i} f_i(\cdot)| \le \omega_i(t) + CM^q\bigg(\|\varepsilon\| + \sum_{l=1}^{n} |M^{-l} x_l|\\ + \sum_{l=1}^{n} |M^{-l} x_l(t-d)|\bigg), \end{aligned}$$

where $C > 0$ and $0 \le q < 1$ are constants.

Assumption 3 *The disturbances $w_i(t), i = 1,\ldots,n$ and their derivatives are bounded.*

Let $M \ge 1$ be a constant. To design the controller $u(t)$, introduce the coordinate transformations

$$\zeta_i = x_i/M^i, \qquad v = u/M^{n+1}. \tag{2}$$

Define $\bar{f}_i = \frac{f_i(\cdot)}{M^i}$, $\bar{\omega}_i = \frac{\omega_i}{M^i}$. By using (2), the x-subsystem can be transformed into

$$\begin{aligned} \dot{\varepsilon} &= f_0\big(t,\varepsilon,x,x(t-d)\big),\\ \dot{\zeta}_i &= Ma_i\zeta_{i+1} + \bar{f}_i + \bar{\omega}_i,\ i=1,\ldots,n-1,\\ \dot{\zeta}_n &= Ma_n v + \bar{f}_n + \bar{\omega}_n. \end{aligned} \tag{3}$$

Next, we need the transformations given in Table 1. The constants $\beta_1, \beta_2, \ldots, \beta_n > 0$ are to be specified later.

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$\xi_1 = \zeta_1$	$\alpha_2 = -\beta_1\xi_1$
$\xi_2 = \zeta_2 - \alpha_2$	$\alpha_3 = -\beta_2\xi_2$
$\vdots$	$\vdots$
$\xi_n = \zeta_n - \alpha_n$	$\alpha_{n+1} = -\beta_n\xi_n$

TABLE I
THE COORDINATE TRANSFORMATIONS.

To design the controller, we provide the following propositions whose proof are omitted due to space limitation.

Lemma 1 *For the auxiliary system*

$$\begin{aligned} \dot{\varepsilon} &= f_0(\cdot), \\ \dot{\zeta}_i &= Ma_i\zeta_{i+1},\ i = 1, \ldots, n-1, \\ \dot{\zeta}_n &= Ma_n v, \end{aligned} \tag{4}$$

we can find a function $V_n(\zeta)$, a control input $v(\zeta)$ and a constant S_1 such that

$$\begin{aligned} \dot{V}_n \leq& -c_0 l_1\|\varepsilon\|^2 - M\sum_{j=1}^{n} a_j H_j \xi_j^2 \\ &+\frac{M^q}{2}e^{-d}S_1\xi_1^2(t-d), \end{aligned} \tag{5}$$

where $V_n(\zeta)$ and $v(\zeta)$ are defined as

$$\begin{aligned} V_n &= c_0 U_0 + \frac{1}{2}\sum_{i=1}^{n}\xi_1^2, \\ v &= -\beta_n\xi_n = -\sum_{j=1}^{n}\Big(\prod_{k=j}^{n}\beta_k\Big)x_j. \end{aligned} \tag{6}$$

Lemma 2 *For $k = 2, \ldots, n$, there exists an arbitrarily small constant π_j and appropriate constants $S_j, \bar{S}_j$ such that*

$$\begin{aligned} &\sum_{k=1}^{n}\xi_k\Big(f_k - \sum_{s=1}^{k-1}\Big(\prod_{l=s}^{k-1}\beta_l\Big)f_s\Big) \\ &\leq \frac{M^q}{2}e^{-d}S_1\xi_1^2(t-d) + M^q e^{-d}\sum_{j=2}^{n}S_j\xi_j^2(t-d) \\ &\quad + M^q\sum_{j=1}^{n}a_j\bar{S}_j\xi_j^2 + \sum_{j=1}^{n}\pi_i\omega_i^2 + \frac{n}{n+1}c_0 l_1\|\varepsilon\|^2. \end{aligned}$$

Now, we summarize the work as follows.

Theorem 1 If Assumptions 1-3 are satisfied, then system (1) has a delay independent controller

$$u(t) = -M^{n+1}\sum_{j=1}^{n}\Big(\prod_{k=j}^{n}\beta_k\Big)x_j. \tag{7}$$

such that the states $\varepsilon, x_1, \ldots, x_n$ are bounded.

Proof: By using the transformations (2), system (1) is tranformed into (3). We choose $U_n = V_n + \sum_{j=1}^{i}W_i, W_i = M^q\int_{t-d}^{t}e^{s-t}S_j\xi_i^2(s)\mathrm{d}s$, it yields that the derivative of U_n along system (3) satisfies

$$\begin{aligned} \dot{U}_n \leq& -c_0 l_1\|\varepsilon\|^2 - M\sum_{k=1}^{n}a_k H_k\xi_k^2 + \frac{M^q}{2}e^{-d}S_1\xi_1^2(t-d) \\ &-\sum_{k=1}^{n}W_k + \sum_{j=1}^{n}\xi_j\Big(f_j - \sum_{l=1}^{j-1}\Big(\prod_{k=l}^{j-1}\beta_k\Big)f_l\Big) \\ &+M^q\sum_{j=1}^{n}S_j\xi_j^2 - M^q e^{-d}\sum_{j=2}^{n}S_j\xi_j^2(t-d). \end{aligned} \tag{8}$$

According to Lemma 2 and (8), one can conclude that

$$\begin{aligned} \dot{U}_n \leq& -\frac{1}{n+1}c_0 l_1\|\varepsilon\|^2 - M\sum_{j=1}^{n}a_j H_j\xi_j^2 - \sum_{i=1}^{n}W_i \\ &+M^q\sum_{j=1}^{n}S_j\xi_j^2 + M^q\sum_{j=1}^{n}a_j\bar{S}_j\xi_j^2 \\ &+\sum_{j=1}^{n}\pi_i\omega_i^2. \end{aligned} \tag{9}$$

Noting that $a_j \geq 1$, it follows that

$$\begin{aligned} \dot{U}_n \leq& -\frac{1}{n+1}c_0 l_1\|\varepsilon\|^2 - M^q\sum_{j=1}^{n}a_j\Big(M^{1-q}H_j \\ &-(S_j+\bar{S}_j)\Big)\xi_j^2 - \sum_{i=1}^{n}W_i + \sum_{j=1}^{n}\pi_i\omega_i^2. \end{aligned} \tag{10}$$

Choosing $M \geq \big(\frac{H_j - 1}{S_j + \bar{S}_j}\big)^{1/(1-q)}$, one has

$$\begin{aligned} \dot{U}_n \leq& -\frac{c_0 l_1}{n+1}\|\varepsilon\|^2 - \sum_{k=1}^{n}a_k\xi_k^2 - \sum_{k=1}^{n}W_k \\ &+\sum_{k=1}^{n}\pi_k\omega_k^2. \end{aligned} \tag{11}$$

Furthermore, by Assumption 1, one gets

$$-\frac{c_0 l_1}{n+1}\|\varepsilon\|^2 \leq -\frac{c_0 l_1}{c_2(n+1)}U_0. \tag{12}$$

Thus, substituting (12) into the inequality (11), one obtains

$$\begin{aligned} \dot{U}_n &\leq -\frac{c_0 l_1}{c_2(n+1)}U_0 - \sum_{j=1}^{n}W_j - \sum_{j=1}^{n}\xi_j^2 + \sum_{j=1}^{n}\pi_j\omega_j^2 \\ &\leq -\rho_1 U_n + \rho_2, \end{aligned} \tag{13}$$

where parameters $\rho_1 = \min\{1, \frac{l_1}{c_2(n+1)}\}$, $\rho_2 = \sum_{j=1}^{n}\pi_j\omega_j^2$.

Considering Assumption 1 and $W_j \geq 0$, we can find $\alpha_1(\cdot) \in \mathcal{K}_\infty$ and obtain that

$$\begin{aligned} U_n &= c_0 U_0 + \sum_{k=1}^{n}W_k + \frac{1}{2}\sum_{k=1}^{n}\xi_k^2 \\ &\geq c_0 c_1\|\varepsilon\|^2 + \frac{1}{2}\sum_{k=1}^{n}\xi_k^2 \geq \alpha_1(\|Z\|), \end{aligned} \tag{14}$$

where $Z = [\varepsilon^\top, \xi_1, \ldots, \xi_n]^\top$. Then, $U_n(Z, t)$ is lower bounded. Next, we will find the upper bound of $U_n(Z, t)$. Due to Assumption 1, we have

$$\begin{aligned} c_0 U_0 + \frac{1}{2}\sum_{j=1}^{n}\xi_j^2 \leq& c_0 c_2\sup_{-d\leq s\leq 0}\|\varepsilon(t+s)\|^2 \\ &+\sup_{-d\leq s\leq 0}\Big(\frac{1}{2}\sum_{j=1}^{n}\xi_j^2(t+s)\Big). \end{aligned} \tag{15}$$