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Traveling waves for a reaction–diffusion–advection predator–prey model

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ABSTRACT

In this paper we study a reaction–diffusion–advection predator–prey model in a river. The existence of predator-invasion traveling wave solutions and prey-spread traveling wave solutions in the upstream and downstream directions is established and the corresponding minimal wave speeds are obtained. While some crucial improvements in theoretical methods have been established, the proofs of the existence and nonexistence of such traveling waves are based on Schauder's fixed-point theorem, LaSalle's invariance principle and Laplace transform. Based on theoretical results, we investigate the effect of the hydrological and biological factors on minimal wave speeds and hence on the spread of the prey and the invasion of the predator in the river. The linear determinacy of the predator–prey Lotka–Volterra system is compared with nonlinear determinacy of the competitive Lotka–Volterra system to investigate the mechanics of linear and nonlinear determinacy.

1. Introduction

Various species and organisms live in media with a predominated unidirectional flow such as rivers and streams. Mathematical models, such as partial differential equations and integro-differential or integro-difference equations have been established to study the dynamics of populations of one or more species in streams or rivers (see e.g., [1–12]). One of the main goals is to understand how populations can persist in such a habitat when continuously subjected to a unidirectional flow (i.e., the "drift paradox") and how the water flow influences spatial population spread and persistence. The studies also provide water management strategy for maintaining desired levels of ecosystem in rivers (i.e., the "instream flow needs"). Predator–prey systems are important components of ecosystems in river or stream environments. It is crucial to understand

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how the interacting preys and predators persist or spread when they are subjected to advective flows. Existing studies of predator–prey systems in rivers are few (see e.g., [3]) and have not theoretically analyzed the effect of the flow on persistence and spread (or invasion) of predator–prey systems.

In this paper we will study the following reaction–diffusion–advection predator–prey system:

$$\begin{cases} u_t = d_1 u_{xx} - b_1 u_x + u(r_1 - b_{11}u - b_{12}v), \\ v_t = d_2 v_{xx} - b_2 v_x + v(r_2 - b_{21}v + b_{22}u), \end{cases} \tag{1}$$

where $u(x,t)$ and $v(x,t)$ denote the densities of the prey and the predator at position $x \in \mathbb{R}$ and time t, respectively, d_1 and d_2 are diffusion constants of the prey and the predator, respectively, b_1 and b_2 are the advection rates of the prey and the predator, respectively, r_1 and r_2 are the growth rates of the prey and the predator, respectively, b_{11} and b_{21} and the density-dependent constants of the prey and the predator, respectively, b_{12} is the predation rate, b_{22} is the conversion rate. Parameters d_1, d_2, r_1 and b_{ij} $(i,j = 1,2)$ are positive constants; b_1, b_2 and r_2 are real numbers. When b_1 and b_2 have different signs, it means that the advective directions of the two species are not the same. For instance, if $b_2 > 0$ represents that the predator is subjected to the flow running from the left (upstream) to the right (downstream), then $b_1 < 0$ represents that the prey such as mayflies flies from the right (downstream) to the left (upstream). If r_2 is positive, then the predator has food sources other than the prey and can grow without the prey; if r_2 is negative, then r_2 essentially is a death rate of the predator and the predator only grows via consuming the prey.

Traveling wave solutions of partial differential equations have attracted increasing interest in recent years (see e.g., [13–30]). A traveling wave solution is a solution of the form

$$u(x,t) = \mathbf{U}(s), \qquad v(x,t) = \mathbf{V}(s), \qquad s = x + \tilde{c}t, \tag{2}$$

where $\tilde{c}$ is the wave speed. It describes the translation of an invariant wave profile in the same direction at a constant speed. For biological models, traveling wave solutions can also show the invasion of species in the spatial habitat.

Various methods have been developed and applied to prove the existence of traveling wave solutions of different types of systems. The monotonic iteration method is powerful for monotonic systems [15,16]. The shooting method, proposed by Dunbar [13,14] and developed by Huang [17], is widely applied to prove the existence of traveling wave solutions for nonmonotonic systems [18–22]. The Schauder's fixed-point theorem is also frequently used to prove the existence of traveling wave solutions for nonmonotonic systems [23–27]. Recently, Huang [31] proposed a geometric approach for some classes of nonmonotonic reaction–diffusion systems; Zhang et al. [30] and Fu et al. [32] developed methods based on the Schauder's fixed-point theorem for nonmonotonic reaction–diffusion systems. The Harnack Inequality method was introduced into traveling waves to study the boundedness of traveling waves by Ducrot and Langlais [33]. However, different advection rates of the prey and the predator have not been considered in the theoretical studies of the minimal wave speeds of traveling waves for nonmonotonic systems. The effect of the water flow on persistence and extinction of predator–prey systems in rivers has been investigated in [3] by virtue of the approximation of traveling wave speeds for decoupled systems, but the theories of the spreading speeds or the minimal wave speeds of traveling waves of predator–prey systems in rivers have not been rigorously established due to the difficulty in mathematical analysis of nonmonotonic systems.

Model (1) is nonmonotonic and the advection coefficients b_1 and b_2 may be different. To derive the necessary and sufficient conditions for the existence of traveling wave solutions of model (1), we extend the methods in [30] for model (1) since different advection coefficients may result in a negative minimal wave speed, which can cause difficulties for the applications of the methods in [30]. Firstly, the upper–lower solutions in [30] must be improved to deal with the negative wave speed. Secondly, the non-triviality or positivity of traveling wave solutions with minimal wave speed for predator–prey models was rarely considered (e.g. [32,19,21,14,18,20]). This positivity of traveling wave solutions with minimal wave speed was confirmed