Nonlinear Analysis: Real World Applications 38 (2017) 21–34

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

Low Mach number limit of a compressible micropolar fluid model

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ARTICLE INFO

Article history:
Received 6 May 2015
Accepted 10 April 2017

Keywords:
Compressible micropolar fluids
Incompressible micropolar fluids
Low Mach number limit
Nonlinear energy method

ABSTRACT

This paper is devoted to investigating the low Mach number limit of a compressible micropolar fluid model. For the well-prepared initial data, we prove rigorously that the solutions of the compressible micropolar fluids converge to that of the incompressible micropolar fluids as the Mach number tends to zero.

1. Introduction

Due to its physical importance, mathematical complexity and wide range of applications, there are many studies on the micropolar fluids, for example, see [1–15], the monograph [16] and the references cited therein. As it is well known that the micropolar fluids were firstly introduced by Eringen [6] in 1966, which is a significant step toward generalization of the classical Navier–Stokes model. From a viewpoint of model, Navier–Stokes models can be viewed as a simplified micropolar fluids. In every point of the micropolar fluids, there is not only the ordinary velocity, but also the microrotational velocity, due to the intrinsic rigid rotation of material elements. From this microscopic view, we can treat these fluids as uniform fluids different from the Newtonian one. Therefore, they are distinguished with the classical fluids.

The general equations which govern the motions of the compressible, viscous, heat conductive micropolar fluids can be described as follows (for example, see, [7,16]):

$$\dot{\rho} + \rho \operatorname{div} \mathbf{u} = 0, \tag{1.1}$$

$$\rho \dot{\mathbf{u}} = \operatorname{div} \mathbf{T} + \rho \mathbf{f}, \tag{1.2}$$

$$\rho j_I \dot{\omega} = \operatorname{div} \mathbf{C} + \mathbf{T}_x + \rho \mathbf{g}, \tag{1.3}$$

$$\rho \dot{E} = \mathbf{T} : \nabla \mathbf{u} + \mathbf{C} : \nabla \omega - \mathbf{T}_x \cdot \omega - \operatorname{div} \mathbf{q} + \rho \delta, \tag{1.4}$$

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http://dx.doi.org/10.1016/j.nonrwa.2017.04.005
1468-1218/

where $\dot{\mathbf{a}}$ is the material derivative of a field $\mathbf{a}$:

$$\dot{\mathbf{a}} = \frac{\partial \mathbf{a}}{\partial t} + (\nabla \mathbf{a}) \cdot \mathbf{u}.$$

And $\mathbf{A} : \mathbf{B}$ denotes the scalar product of tensors $\mathbf{A}$ and $\mathbf{B}$

$$\mathbf{A} : \mathbf{B} = \mathrm{tr}(\mathbf{A}^T \mathbf{B}).$$

The unknowns $\rho > 0, \mathbf{u} = (\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3) \in \mathbb{R}^3, \omega = (\omega_1, \omega_2, \omega_3) \in \mathbb{R}^3, \mathbf{q} \in \mathbb{R}^3, \mathbf{f} \in \mathbb{R}^3, \mathbf{g} \in \mathbb{R}^3, j_I, \delta$ represent the mass density, the velocity, the microrotation velocity, the heat flux density vector, the body force density, the body couple density, the microinertia density(a positive scalar field), the body heat density, respectively, where $\mathbf{q} = -k\nabla\theta$ and θ is the absolute temperature. The dynamic Newtonian viscosity coefficient μ, the second viscosity coefficient λ, the dynamic microrotation viscosity coefficient μ_r, the angular viscosity coefficients c_0, c_d, c_a and the heat conduction coefficient κ are assumed to be constants such that

$$\begin{aligned} &\mu \geq 0, \qquad 3\lambda + 2\mu \geq 0, \qquad \mu_r \geq 0, \\ &c_d \geq 0, \qquad 3c_0 + 2c_d \geq 0, \qquad |c_d - c_a| \leq c_d + c_a, \qquad \kappa \geq 0. \end{aligned}$$

The micropolar fluid is a polar, isotropic fluid when the stress tensor $\mathbf{T}$ and couple stress tensor $\mathbf{C}$ are given by:

$$\mathbf{T} = (-p + \lambda \mathrm{div}\, \mathbf{u})\mathbb{I}_3 + 2\mu \mathrm{sym}(\nabla \mathbf{u}) - 2\mu_r \mathrm{skw}(\nabla \mathbf{u}) - 2\mu_r \omega_{\mathrm{skw}}, \tag{1.5}$$

$$\mathbf{C} = c_0(\mathrm{div}\, \omega)\mathbb{I}_3 + 2c_d \mathrm{sym}(\nabla \omega) - 2c_a \mathrm{skw}(\nabla \omega). \tag{1.6}$$

Here $\mathbb{I}_3$ is a 3×3 unit vector, and $\mathrm{sym}(\mathbf{T}) = \frac{1}{2}(\nabla \mathbf{T} + \nabla \mathbf{T}^T)$, $\mathrm{skw}(\mathbf{T}) = \frac{1}{2}(\nabla \mathbf{T} - \nabla \mathbf{T}^T)$. $\mathbf{T}_x$ is an axial vector with the Cartesian components $\mathbf{T}_x = \epsilon_{ijk} \mathbf{T}_{jk}$, where ϵ_{ijk} is Levi-Civita alternating tensor, and ω_{skw} is the skew tensor with Cartesian components $(\omega_{\mathrm{skw}})_{ij} = \epsilon_{kij}\omega_k$.

From the mathematical point of view, the system (1.1)–(1.6) is too complicated to obtain satisfactory results. Therefore, most of mathematical results are focused on its simplified version. There are two cases: (1) ignore the heat conductivity in the system and reduce (1.1)–(1.6) to the barotropic case, see [3,4], e.c; (2) consider the flow is incompressible, i.e. consider the following system (1.19)–(1.21) (ignore the body force).

The development of mathematical theory for the incompressible micropolar fluids is rather satisfactory, for example, see in [8,16,14,15]. Particularly, the book [16] has elaborated various mathematical analysis in detail. Furthermore, there are still many progresses on the incompressible micropolar fluids recently, for instance, [12] showed that under certain smallness assumption on the rate of change of the initial data and external, there exists a unique and strong solution for any finite time.

On the other hand, there are also a number of works for the compressible micropolar fluids. For instance, [7] showed that for smooth enough spherically symmetric initial data, the compressible micropolar fluids have generalized solution locally in the domain to be the subset of $\mathbb{R}^3$, but the result need the restriction on positive initial density and positive temperature. The same and similar models in one dimensional case were dealt with in [9–11]. The authors [2,3] have shown a blowup criterion of strong solutions to the compressible, viscous micropolar fluids in bounded domains and in the whole space $\mathbb{R}^3$. However, there is no results on the strong solutions of compressible micropolar fluids on the whole space without any restriction. Furthermore, the compressible micropolar fluids is more complex and significant than the incompressible case.

Thus, it is a natural problem to consider the relationship between the solution of incompressible micropolar fluids and that of compressible micropolar fluids. In this paper we shall establish the convergence of the solutions of the compressible micropolar fluids to that of the incompressible micropolar fluids as the Mach number goes to zero.