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The invariant region for the special gas dynamics system☆

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ABSTRACT

In this paper, we study the invariant region for the special gas dynamics system, where the conservation law of energy in classical Euler equations is replaced by the specific entropy. Firstly, the Riemann solutions for five domains in the u–p plane are completely constructed. Then we obtain the bounded convex invariant region of the Riemann solutions through analyzing structure of the Riemann solutions. Moreover, with the bounded convex invariant region, we get the uniform boundedness of approximate solutions, which is the foundation to prove the existence of weak solutions for the special gas dynamic system using compensated compactness method.

1. Introduction

In this paper, we are concerned with equations written in the following conservative form

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + p)_x = 0, \\ (\rho s)_t + (\rho u s)_x = 0, \end{cases} \tag{1}$$

with the Riemann initial data

$$(\rho, \rho u, \rho s)|_{t=0} = \begin{cases} (\rho_r, \rho_r u_r, \rho_r s_r), & x > 0, \\ (\rho_l, \rho_l u_l, \rho_l s_l), & x < 0, \end{cases} \tag{2}$$

where the unknowns $\rho(x,t)$, $u(x,t)$ and $s(x,t)$ denote the density, velocity and entropy, respectively. Here, the pressure p takes the form

$$p(\rho) = k e^{s/c_v} \rho^\gamma, \quad \gamma > 1,$$

where k is a constant with the form of $(\gamma - 1)^2/4\gamma$, and c_v is the specific volume.

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The above equations are equivalent to the compressible Euler equations for smooth solutions which are shown as following

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + p)_x = 0, \\ (\rho E)_t + (\rho u E + up)_x = 0, \end{cases} \tag{3}$$

where E is the energy. The conservation laws of mass, momentum in Eqs. (1) are the same as compressible Euler equations (3), and only the conservation law of the specific entropy replaces that of the energy. In the classical formulations of the gas dynamics system, the total energy conserves and the entropy decreases, while in the special gas system, the entropy conserves and the energy decreases. Even though the classical smooth solutions for both systems are the same, the weak solutions are distinct. The special gas dynamics equations have the physical meaning which are derived from the relaxed model for an isentropic two-phase mixtures [1]. So far, for the full gas dynamics system, the uniform boundedness of approximate solutions has not been obtained, thus, it is not easy to get the existence of weak solutions using compensated compactness method as isentropic Euler equations. The main research results for full gas dynamics are about the comparison with isentropic gas dynamics, see [2–4].

The existence of weak entropy solutions for isentropic Euler equations is concerned actively. Using the vanishing artificial viscosity, Diperna [5] established the existence of the weak entropy solutions for isentropic Euler equations with general L^∞ initial data for $\gamma = 1 + \frac{2}{2n+1}, n \geq 2$ and n is integer. Ding, Chen and Luo [6] also got the existence of the entropy solutions by vanishing numerical viscosity for $\gamma \in (1, \frac{5}{3}]$. Lions, Perthame and Tadmor [7], and Lions, Perthame and Souganidis [8] got the existence results for $\gamma \geq 3$ and $1 < \gamma < 3$, respectively. Huang and Wang [9] got the existence result for $\gamma = 1$. So far, the existence of weak entropy solutions for isentropic Euler equations is perfectly solved. While for the full system, it has few properties as the isentropic gas dynamics system due to the lack of mathematical entropies. Thus it has been recognized that the compensated compactness method should fail in general.

With the vanishing artificial viscosity, Bereux, Bonnetier and Lefloch [1] have studied the existence of weak entropy for the special system using the compensated compactness method. More details can be seen in [10]. However, the uniform boundedness of approximate solutions has not be gotten in [1], where it is assumed to hold. In order to get the uniform boundedness of approximate solutions, it is effective to get the invariant region. Recently, Jiang and Wang [11] proved that the bounded invariant region of Riemann solutions for the full gas dynamics does not exist. In this paper, for the entropy being unchanged across shock waves and rarefaction waves, we obtain the invariant region of the Riemann solutions for the special gas dynamics system where the conservation law of the special entropy replaces that of the energy in the full system. Moreover, we get the uniform boundedness of approximate solutions, which is the basis to get the existence of weak solutions for special gas dynamics system using compensated compactness method as the isentropic Euler equations.

The rest of this paper is organized as follows. In Section 2, we present some preliminary knowledge and basic facts about the system (1). In Section 3, we construct the Riemann solutions in different domains, and get the bounded convex invariant region of the Riemann solutions. Moreover, the uniform boundedness of approximate solutions is obtained.

2. Preliminaries

In this section, we briefly introduce some basic facts about Eqs. (1) with the Riemann initial data.

The eigenvalues of the system (1) are

$$\lambda_1 = u - c, \qquad \lambda_2 = u, \qquad \lambda_3 = u + c,$$