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Global existence and exponential stability for a nonlinear thermoelastic Kirchhoff–Love plate

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ABSTRACT

We study an initial–boundary-value problem for a quasilinear thermoelastic plate of Kirchhoff & Love-type with parabolic heat conduction due to Fourier, mechanically simply supported and held at the reference temperature on the boundary. For this problem, we show the short-time existence and uniqueness of classical solutions under appropriate regularity and compatibility assumptions on the data. Further, we use barrier techniques to prove the global existence and exponential stability of solutions under a smallness condition on the initial data. It is the first result of this kind established for a quasilinear non-parabolic thermoelastic Kirchhoff & Love plate in multiple dimensions.

1. Introduction

Let $\Omega \subset \mathbb{R}^d$ ($d = 2$ or 3) be a smooth bounded domain representing the mid-plane of a thermoelastic plate. With w and θ denoting the vertical deflection and an appropriately weighted thermal moment with respect to the plate thickness, both depending on a scaled time variable $t > 0$ and the space variable $(x_1, x_2) \in \Omega$, the nonlinear Kirchhoff & Love thermoelastic plate system reads as

$$w_{tt} - \gamma \triangle w_{tt} + a(-\triangle w)\,\triangle^2 w + \alpha \triangle \theta = f(-\triangle w, -\nabla \triangle w) \quad \text{in } (0,\infty) \times \Omega, \tag{1.1a}$$

$$\beta \theta_t - \eta \triangle \theta + \sigma \theta - \alpha \triangle w_t = 0 \quad \text{in } (0,\infty) \times \Omega \tag{1.1b}$$

along with the boundary conditions (hinged mechanical/Dirichlet thermal)

$$w = \triangle w = \theta = 0 \quad \text{in } (0,\infty) \times \Omega \tag{1.1c}$$

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and the initial conditions

$$w(0,\cdot) = w^0, \qquad w_t(0,\cdot) = w^1, \qquad \theta(0,\cdot) = \theta^0 \quad \text{in } \Omega. \tag{1.1d}$$

Here, $\alpha, \beta, \gamma, \eta, \sigma$ are positive constants and $a\colon \mathbb{R} \to (0,\infty)$ as well as $f\colon \mathbb{R} \times \mathbb{R}^d \to \mathbb{R}$ are smooth functions. For thin plates, γ behaves like h^2 as $h \to 0$ (cf. [1, Equation (2.16), p. 13]) and is, therefore, neglected in some literature. In Section 2, we present a short physical deduction of Eqs. (1.1a)–(1.1d).

Lasiecka et al. [2] studied a quasilinear PDE system similar to (1.1a)–(1.1d) in a smooth, bounded domain Ω of $\mathbb{R}^d$ with $d \leq 3$ given by a Kirchhoff & Love plate with parabolic heat conduction

$$w_{tt} + \triangle^2 w - \triangle\theta + a\triangle\big((\triangle w)^3\big) = 0 \quad \text{in } (0,T) \times \Omega, \tag{1.2a}$$

$$\theta_t - \triangle\theta + \triangle w_t = 0 \quad \text{in } (0,T) \times \Omega \tag{1.2b}$$

together with boundary conditions (1.1c) and initial conditions (1.1d) for an arbitrary $T > 0$. For the initial–boundary-value problem (1.2a)–(1.2b), (1.1c)–(1.1d), they proved the global existence of weak solutions (w,θ) and their uniform decay in the norm of

$$\Big(W^{1,\infty}\big(0,T;L^2(\Omega)\big) \cap L^\infty\big(0,T;W^{2,4}(\Omega)\big)\Big) \times L^\infty\big(0,T;W^{1,2}(\Omega)\big).$$

The existence proof was based on a Galerkin approximation and compactness theorems, while the uniform stability was obtained with the aid of energy techniques.

In their monograph [3], Chueshov and Lasiecka give an extensive study on the von Kármán plate system both in pure elastic and thermoelastic cases. With $w\colon \Omega \to \mathbb{R}$ denoting the vertical displacement and $v\colon \Omega \to \mathbb{R}$ standing for the Airy stress function of a plate with its mid-plane occupying in the reference configuration a domain $\Omega \subset \mathbb{R}^2$, the pure elastic version of Kármán plate system reads as

$$w_{tt} - \alpha\triangle w_{tt} + \triangle^2 u - [u, v + F_0] + Lu = p \quad \text{in } (0,\infty) \times \Omega, \tag{1.3a}$$

$$\triangle^2 v + [u,u] = 0 \quad \text{in } (0,\infty) \times \Omega, \tag{1.3b}$$

where $[v,w] := v_{x_1x_1}w_{x_2x_2} + v_{x_2x_2}w_{x_1x_1} - 2v_{x_1x_2}w_{x_1x_2}$, L is a first-order differential operator and $F_0, p\colon \Omega \to \mathbb{R}$ are given "force" functions. Imposing standard initial conditions, under various sets of boundary conditions, Chueshov and Lasiecka proved Eqs. (1.3a)–(1.3b) possess a unique generalized, weak or strong solution depending on the data regularity. The proof was based on a nonlinear Galerkin-type approximation. Further, they studied the semiflow associated with the solution to Eqs. (1.3a)–(1.3b), in particular, they analyzed its long-time behavior and the existence of attracting sets. Various damping mechanisms, thermoelastic effects, structurally coupled systems such as acoustic chambers or gas flow past a plate were studied. An extremely detailed and comprehensive literature overview was also given.

Denk et al. [4] considered a linearization of (1.2a)–(1.2b), which corresponds to letting $a \equiv 0$, in a bounded or exterior C^4-domain of $\mathbb{R}^d$ for $d \geq 2$ subject to the initial conditions from Eq. (1.1d) and the boundary conditions

$$w = \partial_\nu w = \theta = 0 \quad \text{on } (0,T) \times \partial\Omega, \tag{1.4}$$

where $\partial_\nu = (\nabla\cdot)^T \nu$ and ν denotes the outer unit normal vector to Ω on $\partial\Omega$. By proving a resolvent estimate both in the whole space and in the half-space and employing localization techniques, they showed that the C_0-semigroup for (w, w_t, θ) on the space

$$W^{2,p}_D(\Omega) \times L^p(\Omega) \times L^p(\Omega) \quad \text{with } W^{2,p}_D(\Omega) = \{u \in W^{2,p}(\Omega) \mid u = \partial_\nu u = 0 \text{ on } \partial\Omega\}$$

is analytic. In case Ω is bounded, they also proved an exponential stability result for the semigroup.

Lasiecka and Wilke [5] presented an L^p-space treatment of Eqs. (1.2a)–(1.2b), (1.1c)–(1.1d) in bounded C^2-domains Ω of $\mathbb{R}^d$. By proving the maximal L^p-regularity for the linearized problem, they adopted the