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Systems of coupled clamped beams equations with full nonlinear terms: Existence and location results[☆]

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ABSTRACT

This work gives sufficient conditions for the solvability of the fourth order coupled system

$$\begin{cases} u^{(4)}(t) = f(t, u(t), u'(t), u''(t), u'''(t), v(t), v'(t), v''(t), v'''(t)) \\ v^{(4)}(t) = h(t, u(t), u'(t), u''(t), u'''(t), v(t), v'(t), v''(t), v'''(t)) \end{cases}$$

with $f, h : [0, 1] \times \mathbb{R}^8 \to \mathbb{R}$ some L^1- Carathéodory functions, and the boundary conditions

$$\begin{cases} u(0) = u'(0) = u''(0) = u''(1) = 0 \\ v(0) = v'(0) = v''(0) = v''(1) = 0. \end{cases}$$

To the best of our knowledge, it is the first time in the literature where two beam equations are considered with full nonlinearities, that is, with dependence on all derivatives of u and v.

An application to the study of the bending of two elastic coupled clamped beams is considered.

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1. Introduction

In this paper we consider the fourth order coupled system

$$\begin{cases} u^{(4)}(t) = f(t, u(t), u'(t), u''(t), u'''(t), v(t), v'(t), v''(t), v'''(t)) \\ v^{(4)}(t) = h(t, u(t), u'(t), u''(t), u'''(t), v(t), v'(t), v''(t), v'''(t)) \end{cases} \tag{1}$$

with $f, h: \ [0,1] \times \mathbb{R}^8 \to \mathbb{R}$ some L^1- Carathéodory functions and the boundary conditions

$$\begin{cases} u(0) = u'(0) = u''(0) = u''(1) = 0 \\ v(0) = v'(0) = v''(0) = v''(1) = 0. \end{cases} \tag{2}$$

Fourth order differential equations have been studied by many authors with different types of boundary conditions, as it can be seen in [1–16] and the references therein. The applications can be found in several fields, such as the description of the motion of the road bed of suspension bridges [17], nonlocal elasticity theory, more precisely the study of nonlinear vibrations of an Euler–Bernoulli nanobeam [18], study of the flow of certain fluids over stretching or shrinking sheets [19], among others.

In engineering, there exist a huge number of problems where a beam is supported on elastic foundations, exerting a reaction on the beam, or nonelastic ones. We refer, as example, the study of the (linear) deflection in [20], applying a perturbation technique to examine the deflection of a general elastically end-restrained, non-uniform beam resting on a non-linear elastic foundation; [21], considering the problem of bending of axially constrained beams on nonlinear Winkler-type elastic foundations, where the solution is obtained by an iteration method using Green's functions; [22], studying the effects of traveling mass with variable velocity on dynamic response regarding an inclined Timoshenko beam.

Boundary value problems composed by systems of fourth order differential equations are more scarce (see, for instance, [23–26]). In [25] the authors consider the existence of multiple positive solutions for coupled singular system of second and fourth order ordinary differential equations

$$\begin{cases} u^{(4)} = f(t, v), \quad (t, v) \in (0,1) \times \mathbb{R}^+, \\ -v'' = g(t, u), \quad (t, u) \in (0,1) \times \mathbb{R}^+, \\ u(0) = u(1) = u''(0) = u''(1) = 0 \\ v(0) = v(1) = 0, \end{cases}$$

where $f \in C[(0,1) \times \mathbb{R}^+, \mathbb{R}^+]$ and $g \in C[(0,1) \times \mathbb{R}^+, \mathbb{R}^+]$, applying a fixed point theorem of cones expansion and compression.

In [26] it is studied the fourth-order nonlinear singular semipositone system

$$\begin{cases} x^{(4)}(t) = f(t, x(t), y(t), x''(t), y''(t)), \\ y^{(4)}(t) = g(t, x(t), y(t), x''(t), y''(t)), \quad t \in (0,1), \\ x(0) = x(1) = x''(0) = x''(1) = 0 \\ y(0) = y(1) = y''(0) = y''(1) = 0, \end{cases}$$

with $f, g \in C((0,1) \times [0,\infty) \times [0,\infty) \times (-\infty, 0] \times (-\infty, 0], \mathbb{R})$, by approximating the fourth-order system to a second-order singular one and using a fixed point index theorem on cones, to guarantee the existence of positive solutions of the problems.

Motivated by the above papers we consider the problem (1), (2), where the nonlinearities can depend on all derivatives of both unknown functions. To the best of our knowledge, this is new in the literature and it opens the possibility of new types of models to study the bending of a system of two coupled beams where different shear forces happen on each beam. Moreover we prove, by a concrete application, that there are cases where the bending of both beams have opposite signs.