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A simplified stationary energy-transport model with temperature-dependent conductivity

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ABSTRACT

A simplified stationary energy-transport model in semiconductors is studied while the conductivity depends on both electron density and temperature. The existence and uniqueness of classical solutions to the model is proved for small variations of electron density and temperature.

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1. Introduction and main results

In this paper, we consider the following simplified energy-transport model for semiconductors (see [1]):

$$\rho_t + \mathrm{div} j = 0, \tag{1.1}$$

$$-\nabla(\rho\theta) + \rho\nabla\phi = j, \tag{1.2}$$

$$-\mathrm{div}(k(\rho,\theta)\nabla\theta) = \frac{\rho}{\tau_e}(\theta_L(x) - \theta), \tag{1.3}$$

$$\lambda^2 \triangle\phi = \rho - D(x), \quad x \in \Omega,\ t > 0, \tag{1.4}$$

where the electron density ρ, the electron temperature θ, the electron current j, and the electric potential ϕ are unknown variables; $\theta_L(x)$ and $D(x)$ denote the lattice temperature and the doping profile, respectively; the energy relaxation time $\tau_e > 0$ and the Debye length $\lambda > 0$ are the scaled physical parameters. The

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conductivity $k(\rho,\theta)$ is usually a function of the electron density and the electron temperature in the semiconductor physics, see, for instance, the function $k(\rho,\theta)=\rho\theta$ was taken in [2,3].

In [4], Ben Abdallah and Degond derived a complicated form of energy transport model from the semiconductor Boltzmann equation. The analysis of this model is very hard due to the strong coupling with temperature gradients and there have been only partial results for it so far. For the energy transport model derived in [4], the existence of stationary solutions near thermal equilibrium was obtained in [5,6], while the existence of transient solutions close to equilibrium was studied in [7–9] and the systems with uniformly positive definite diffusion matrices were considered in [10,11]. Furthermore, the nonlinear stability of classical bounded solutions to the one-dimensional equations was investigated on the whole real line in [12]. However, there is not a complete global existence result of this model for any data and physical transport coefficients. Moreover, the direct solution of the model derived in [4] is very time-consuming even with modern computers. One needs to derive simpler model equations which still contain the most important physical information. Therefore, Jüngel, Pinnau and Röhrig introduced in [1] the simplified energy-transport model (1.1)–(1.4) which still includes temperature gradients but the coupling with the energy equation is weaker than [4]. An important feature of the model (1.1)–(1.4) is that it is derived formally from the hydrodynamic semiconductor equations in a zero relaxation time limit, which provides a physical modeling basis without heuristics [1].

When the thermal conductivity $k(\rho,\theta)$ depends only on the electron density ρ, Jüngel, Pinnau and Röhrig [1] obtained the global-in-time existence of bounded weak solutions to the model (1.1)–(1.4) subject to mixed Dirichlet–Neumann boundary conditions and the authors proved the existence of classical solutions to the stationary 1-dimensional model of (1.1)–(1.4) in [13]. However, there have been very few results for the case that $k(\rho,\theta)=\rho\theta$, at least to the best of our knowledge.

In this paper, our goal is to study the existence and uniqueness of classical solutions to the stationary 1-dimensional model of (1.1)–(1.4) under the condition that $k(\rho,\theta)=\rho\theta$. For simplicity, let $\Omega=(0,1)$, and $\tau_e=\lambda=1$. We consider the following boundary value problem:

$$j_x=0, \tag{1.5}$$
$$-(\rho\theta)_x+\rho\phi_x=j, \tag{1.6}$$
$$-(\rho\theta\theta_x)_x=\rho(\theta_L(x)-\theta), \tag{1.7}$$
$$\phi_{xx}=\rho-D(x),\quad x\in\Omega \tag{1.8}$$
$$\rho(0)=\rho_l>0,\qquad \rho(1)=\rho_r>0, \tag{1.9}$$
$$\theta(0)=\theta_l>0,\qquad \theta(1)=\theta_r>0, \tag{1.10}$$
$$\phi(0)=0,\qquad \phi(1)=\phi_r>0 \tag{1.11}$$

where we assume that

(A1) $\theta_L(x), D(x)\in C^0(\overline{\Omega})$;
(A2) $0<\theta_m\le\theta_L(x)\le\theta_M, 0<D_m\le D(x)\le D_M$, with $\theta_m,\theta_M,D_m,D_M$ being constants.

Comparing with the previous results in [1,13], we study the model with the physical conductivity $k(\rho,\theta)=\rho\theta$. In our situation, Eq. (1.3) becomes quasilinear degenerate due to the influence of temperature on the thermal conductivity. A new fixed-point mapping and a different function set are introduced to deal with this problem. Moreover, more careful analysis is required to estimate the electron density and the electron temperature. In addition, quite different from the arguments in [13] where the electron current was given ($j=j_0$), we will study the quantity of the current for given voltage on the boundary of the domain. We remark that this boundary condition is more reasonable in physical applications. Finally, the uniqueness of classical solutions to the problem (1.5)–(1.11) is also proved for the small variations of electron density and temperature.

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