Nonlinear Analysis: Real World Applications 35 (2017) 75–101

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

Existence of solutions of a thermoviscoplastic model and associated optimal control problems

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HIGHLIGHTS

- We prove the existence of a unique weak solution of a thermoviscoplastic model.
- The model is fully coupled and it contains a variational inequality of second kind.
- We exploit an integrability result for nonl. elasticity and max. parabolic regularity.
- We formulate a class of opt. control problems and show existence of global minimizers.
- We show that the control-to-state operator is weakly sequentially continuous.

ARTICLE INFO

Article history:
Received 17 July 2015
Received in revised form 29 April 2016
Accepted 19 October 2016

Keywords:
Thermoviscoplasticity
Variational inequality of second kind
Mixed boundary conditions
Banach fixed-point theorem
Maximal parabolic regularity
Optimal control

ABSTRACT

A quasistatic, thermoviscoplastic model at small strains with linear kinematic hardening, von Mises yield condition and mixed boundary conditions is considered. The existence of a unique weak solution is proved by means of a fixed-point argument, and by employing maximal parabolic regularity theory. The weak continuity of the solution operator is also shown. As an application, the existence of a global minimizer of a class of optimal control problems is proved.

1. Introduction

We consider the following quasistatic, thermovisco(elasto)plastic model at small strains with linear kinematic hardening and von Mises yield condition:

$$\text{stress–strain relation:} \qquad \boldsymbol{\sigma} = \mathbb{C}\big(\boldsymbol{\varepsilon}(\boldsymbol{u}) - \boldsymbol{p} - \boldsymbol{t}(\theta)\big), \tag{1}$$

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http://dx.doi.org/10.1016/j.nonrwa.2016.10.008
1468-1218/

conjugate forces:	$\boldsymbol{\chi} = -\mathbb{H}\,\boldsymbol{p}$,	(2)
viscoplastic flow rule:	$\epsilon\,\dot{\boldsymbol{p}} + \partial_{\dot{\boldsymbol{p}}} D(\dot{\boldsymbol{p}}, \theta) \ni [\boldsymbol{\sigma} + \boldsymbol{\chi}]$,	(3)
balance of momentum:	$-\operatorname{div}\big(\boldsymbol{\sigma} + \gamma\,\boldsymbol{\varepsilon}(\dot{\boldsymbol{u}})\big) = \boldsymbol{\ell}$,	(4)
heat equation:	$\varrho\, c_p\, \dot{\theta} - \operatorname{div}(\kappa\,\nabla\theta) = r + \gamma\,\boldsymbol{\varepsilon}(\dot{\boldsymbol{u}}) : \boldsymbol{\varepsilon}(\dot{\boldsymbol{u}}) + (\boldsymbol{\sigma} + \boldsymbol{\chi}) : \dot{\boldsymbol{p}} - \theta\boldsymbol{t}'(\theta) : \mathbb{C}(\boldsymbol{\varepsilon}(\dot{\boldsymbol{u}}) - \dot{\boldsymbol{p}})$.	(5)

The unknowns are the stress $\boldsymbol{\sigma}$, back-stress $\boldsymbol{\chi}$, plastic strain $\boldsymbol{p}$, displacement $\boldsymbol{u}$ and temperature θ. Further, $\mathbb{C}$ and $\mathbb{H}$ denote the elastic and hardening moduli, respectively. $\boldsymbol{\varepsilon}(\boldsymbol{u})$ denotes the symmetrized gradient or linearized strain associated with $\boldsymbol{u}$. The temperature dependent term $\boldsymbol{t}(\theta)$ expresses thermally induced strains. D denotes the dissipation function. The right hand sides $\boldsymbol{\ell}$ and r represent mechanical and thermal volume and boundary loads, respectively. ϱ, c_p and κ describe the density, specific heat capacity and thermal conductivity of the material. The positive parameters ϵ and γ represent viscous effects in the evolution of the plastic strain and in the balance of momentum. For the derivation of the system (1)–(5) and more on its physical background, we refer the reader to Ottosen & Ristinmaa, [1, Chapter 22 and 23].

The analysis of thermoplastic models poses numerous mathematical challenges, mainly due to the low integrability of the nonlinear terms on the right hand side of the heat equation. Several approaches have been considered in the literature to deduce the existence and uniqueness of a solution, and we mention the following.

- Chełmiński & Racke [2]: In this model without viscosity terms the dissipation function is only allowed to depend linearly on the temperature and a simplified mechanical heat source is used which does not account for plastic dissipation and is cut off at large temperatures. The authors use a Yosida regularization to prove the existence of a solution.
- Bartels & Roubíček [3]: The model does not account for hardening and thermal strains, it contains a hyperbolic viscous balance of momentum and a simplified right hand side of the heat equation. The authors prove the existence of a solution in a weak sense via a discretization strategy.
- Bartels & Roubíček [4]: In contrast to Bartels & Roubíček [3] the authors take into account thermal strains, linear kinematic and isotropic hardening and the same right hand side of the heat equation as in (5) but they consider a temperature independent flow rule. The authors require a growth condition for the heat capacity w.r.t. the temperature to obtain the existence of a solution in a weak sense, again via a discretization procedure.
- Paoli & Petrov [5]: In contrast to our model the authors assume a C^2 regular boundary in addition to homogeneous boundary conditions for the displacement, which leads to better regularity. Moreover, the dissipation function is assumed to be independent of the temperature. The authors use a growth condition for the heat capacity w.r.t. the temperature to show the existence of a solution in a classical sense by means of Schauder's fixed point theorem.

Our approach is closest to the one in [5]. We emphasize that we admit more general domains and boundary conditions. The overall strategy to show the existence and uniqueness of a solution is an application of Banach's fixed point theorem, applied to a reduced problem formulated in the temperature variable alone. In order to apply the fixed-point argument, we make use of the theory of maximal parabolic regularity. The same strategy was used in [6] for the analysis and optimal control of a thermistor problem. Furthermore, we focus our discussion on the case of constant heat capacities. We mention that this case is not included in [5] since a linear growth of the heat capacity is assumed there. In contrast to the linear dependence of the thermal strain on the temperature in [5], we allow more general thermal strains $\boldsymbol{t}$ and only assume them to be globally bounded w.r.t. the temperature. This can be achieved w.l.o.g. by a cut-off outside the relevant temperature regime.

Under the assumptions made precise in Section 2, our main result is as follows. (We refer the reader to Theorem 10 for a re-iteration of the theorem.)