



Reaction enhancement by chemotaxis



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ABSTRACT

An interesting problem arising in many contexts of mathematical biology is the study of the relevance of chemotaxis in reaction–diffusion processes. We approach this problem through a mathematical model representing the fertilization process of corals. As a result we obtain a system of partial differential equations describing the cell dynamics being affected by three basic phenomena: *diffusion*, *chemotaxis* and *a surrounding flux*. First we prove that our model has in general global classical solutions. Next, we compare the asymptotic behavior with and without chemotaxis. We show the relevance of chemotaxis after making a systematic adaptation of the well-known moments technique on bounded domains (usually used for proving blow-up in Keller–Segel systems) to analyze the behavior of the cell dynamics when the chemotactic signal increases. In the context of the proposed model, our analysis shows that for suitable initial data, the remaining fraction of unfertilized eggs at any given time $\tau > 0$ becomes arbitrarily small if the chemotaxis signal is sufficiently large and the sperm and eggs are concentrated enough at the initial time.

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1. Introduction

What mechanisms allow sperm cells to find eggs? An interesting example is coral spawning. It has been shown that fertilization occurs 90% of the time [1]. We know that chemotaxis takes place in instances where there is a high rate of fertilization [2–5]. Some researchers [6,7] have approached this fertilization process from a mathematical point of view, showing through some mathematical models that without chemotaxis not all the eggs are fertilized, while with chemotaxis all eggs are ultimately fertilized. However, their models unrealistically assumed that eggs and sperm have exactly the same concentration during the whole process. An attempt to create a more accurate model, having global solutions in bounded and unbounded domains, was made recently [8]. This last proposal has been further developed for the case of bounded domains [9,10].

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The aim of this paper is to develop a new and more accurate model by eliminating the assumption that eggs and sperm have the same densities (e.g. [8,6,7]). We take into account that the sperm cells are following a chemical signal, whereas the eggs are not. To account for this, we add a reaction–diffusion equation modeling the egg concentration. We consider the case where the cells and eggs do not affect the velocity of the fluid \mathbf{u} , which we simply take as given. This results in a Keller–Segel type model

$$\begin{aligned} c_t + \mathbf{u} \cdot \nabla c &= d_0 \Delta c - ac + k_0 e, \\ \rho_t + \mathbf{u} \cdot \nabla \rho &= d_1 \Delta \rho - \chi \nabla \cdot (\rho \nabla c) - k_1 \rho e, \\ e_t + \mathbf{u} \cdot \nabla e &= d_2 \Delta e - k_1 \rho e. \end{aligned} \tag{1}$$

The unknowns are the density of sperm ρ that have not fertilized an egg, the density e of unfertilized eggs and the concentration c of the chemical expelled by the eggs to attract sperm. The parameters $k_0, k_1, d_0, d_1, d_2, a$ and χ represent positive nonzero constants. We observe that we can simplify the system through the transformations $e \mapsto k_1 e$ and $\rho \mapsto k_1 \rho$, which bring us to assume through the whole paper that $k_1 = 1$.

We will approach in this paper a simplified version of model (1) assuming that the Péclet number for the chemical concentration is small, which allows us to ignore the effects of convection for the chemical concentration. On the other hand, this assumption also means that *we are not considering the possibility of having fast fluid flows*. We also assume that the diffusion of the chemical is much faster than that of eggs and cells. These hypotheses lead us to the next simplified version of the model (1),

$$\left. \begin{aligned} 0 &= \Delta c + k_0 \left(e - \frac{1}{|\Omega|} \int_{\Omega} e dx \right), \quad \text{with } \int_{\Omega} c dx = 0 \\ \rho_t + \mathbf{u} \cdot \nabla \rho &= d_1 \Delta \rho - \chi \nabla \cdot (\rho \nabla c) - \rho e, \quad x \in \Omega, \quad t > 0, \\ e_t + \mathbf{u} \cdot \nabla e &= d_2 \Delta e - \rho e, \quad x \in \Omega, \quad t > 0. \end{aligned} \right\} \tag{2}$$

In the case of coral reproduction, dispersion phenomena could play an important role in the biological description of the whole process, and therefore we propose to study our system not only in bounded domains but also in unbounded domains. In this paper, however, we consider only a bounded domain, which we denote by Ω .

We assume homogeneous Neumann conditions, i.e.,

$$\frac{\partial c}{\partial \nu} = \frac{\partial \rho}{\partial \nu} = \frac{\partial e}{\partial \nu} = 0 \quad \text{on } \partial \Omega. \tag{3}$$

We take positive initial data and denote it by

$$\rho(x, 0) = \rho_0, \quad e(x, 0) = e_0, \quad x \in \Omega. \tag{4}$$

For the fluid velocity, we assume boundedness, smoothness and incompressibility, more precisely

$$\sup_{t \in [0, \infty)} \|\mathbf{u}(\cdot, t)\|_{L^\infty(\Omega)} < \infty, \quad \mathbf{u} \in C^1(\overline{\Omega} \times [0, \infty), \mathbb{R}^n) \quad \text{and} \quad \nabla \cdot \mathbf{u} = 0. \tag{5}$$

We also assume the no-slip boundary condition

$$\mathbf{u} = 0 \quad \text{on } \partial \Omega. \tag{6}$$

In Section 2 we prove that the model (2)–(6) in general has global classical solutions in the two-dimensional case. This result is achieved through a careful application of the theory of regularity together with some estimates for the Neumann heat semigroup $(e^{t\Delta})_{t \geq 0}$ under Neumann boundary conditions. Next, we approach in Section 3 the problem of asymptotic behavior. We show that if χ is small enough and velocity of the fluid is low, then the assumption

$$\int_{\Omega} \rho_0 dx \geq \int_{\Omega} e_0 dx \tag{7}$$

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