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A magneto-viscoelasticity problem with a singular memory kernel

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A B S T R A C T

The existence of solutions to a one-dimensional problem arising in magneto-viscoelasticity is here considered. Specifically, a non-linear system of integro-differential equations is analysed; it is obtained coupling an integro-differential equation modelling the viscoelastic behaviour, in which the kernel represents the relaxation function, with the non-linear partial differential equations modelling the presence of a magnetic field. The case under investigation generalizes a previous study since the relaxation function is allowed to be unbounded at the origin, provided it belongs to L^1; the magnetic model equation adopted, as in the previous results (Carillo et al., 2011, 2012; Chipot et al. 2008, 2009) is the penalized Ginzburg–Landau magnetic evolution equation.

1. Introduction

The study of magneto-viscoelastic materials is motivated by the interest on mechanical properties of innovative materials widely studied in a variety of applications. In particular, as far as the coupling between mechanical and magnetic effects is concerned, the interest is motivated by new materials such as Magneto Rheological Elastomers or, in general, magneto-sensitive polymeric composites (see [1,2] and references therein). A variational approach to study multiscale models, in this context, is given in [3]. The results here presented are connected to a wide research project concerning the analytical study of differential and integro-differential models connected to mechanical properties of materials. Thus, in [4–6] magneto-elasticity problems are considered, in [7,8] magneto-viscoelasticity problems are studied. Then, in turn, the case of a 1-dimensional, and of a 3-dimensional, body is investigated under the assumption of a regular kernel representing the relaxation modulus. Later, materials with memory characterized by a singular kernel integro-differential equations are studied in [9–12]. Indeed, as pointed out therein, the case is of interest

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not only to model different physical behaviours but also under the analytical viewpoint. The interest in non-classical memory kernels and, in particular, in *singular kernel* problems, as pointed out also in [12], goes back to Boltzmann [13] and later, is testified, analytically, by the results of Berti [14], Gentili [15], Giorgi and Morro [16], Grasselli and Lorenzi [17] and Hanyga et al. [18–20]. In addition, *fractional derivative* models, since the works of Rabotnov [21] and Koeller [22], are employed in [23–25]. Here, a viscoelastic body is studied under the assumption of a relaxation modulus, modelled by a L^1 function, coupled with a magnetic field.

The problem to study is concerned with the behaviour of a viscoelastic body subject also to the presence of a magnetic field. The body is assumed to be one-dimensional. In particular the problem under investigation is motivated by a great interest in the realization of new materials which, on one side couple a viscoelastic behaviour with a magnetic one, see

$$\begin{cases} u_t(t) - \displaystyle\int_0^t G(t-\tau)u_{xx}(\tau)d\tau - u_1 - \int_0^t \frac{\lambda}{2}(\Lambda(\mathbf{m})\cdot\mathbf{m})_x d\tau \ = \int_0^t f(\tau)d\tau \\ \mathbf{m}_t + \mathbf{m}\dfrac{|\mathbf{m}|^2-1}{\delta} + \lambda\Lambda(\mathbf{m})u_x - \mathbf{m}_{xx} = 0, \end{cases} \quad \text{in } \mathcal{Q} \tag{1}$$

together with the initial and boundary conditions

$$u(\cdot,0) = u_0 = 0, \qquad \mathbf{m}(\cdot,0) = \mathbf{m}_0, \qquad |\mathbf{m}_0| = 1 \quad \text{in } \Omega, \tag{2}$$

$$u = 0, \qquad \frac{\partial \mathbf{m}}{\partial \boldsymbol{\nu}} = 0 \quad \text{on } \Sigma = \partial\Omega \times (0,T), \tag{3}$$

where $\Omega = (0,1)$, $\mathcal{Q} := \Omega \times (0,T)$ and $\mathcal{M} \equiv (0,\mathbf{m})$, letting $\mathbf{m} = (m_1, m_2)$, is the magnetization vector, orthogonal to the conductor so that, since $\mathbf{u} \equiv (u,0,0)$, when both quantities are written in $\mathbb{R}^3$; in addition, $\boldsymbol{\nu}$ is the outer unit normal at the boundary $\partial\Omega$, Λ is a linear operator defined by $\Lambda(\mathbf{m}) = (m_2, m_1)$, the scalar function u is the displacement in the direction of the conductor itself, here identified with the x-axis and λ is a positive parameter. In addition, the term f represents an external force which also includes the deformation history.

Moreover we assume:

$$u_1 \in L^2(\Omega), \qquad \mathbf{m}_0 \in \mathbf{H}^1(\Omega), \qquad f \in L^2(\mathcal{Q}). \tag{4}$$

The model adopted here to describe the magneto-elastic interaction is introduced in [26,4–6] and the case of magneto-viscoelastic regular behaviour is given in [7,8].

In fact, the kernel in the linear integro-differential equation, which represents the relaxation function G, is assumed here to satisfy weaker functional requirements with respect to the *classical* regularity requirements. In particular, the relaxation function $G(t)$ is assumed to be such that

$$G \in L^1(0,T) \cap C^2(0,T), \quad \forall\, T \in \mathbb{R}^+; \tag{5}$$

the relaxation function $G(t)$ is assumed to satisfy the further requirements, which follow from the physics of the model,

$$G(t) > 0, \qquad \dot{G}(t) \le 0, \qquad \ddot{G}(t) \ge 0, \quad t \in (0,\infty). \tag{6}$$

Note that, in the *classical* model the relaxation function, further to satisfy conditions (6), is assumed to be $C^2[0,T]$, $\forall T \in \mathbb{R}^+$. To this aim, in the following Section 2, a suitable sequence of approximated classical problems is constructed. In the same Section also some a priori estimates are obtained. Crucial in our analysis is the assumption $u_0 = 0$.

The subsequent Section 3, is devoted to prove the existence of a weak solution to the problem (1) with the initial and boundary conditions (2)–(3).