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Existence and stability of coexistence states in a competition unstirred chemostat

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ABSTRACT

In this paper, we consider the two similar competing species in a competition unstirred chemostat model with diffusion. The two competing species are assumed to be identical except for their maximal growth rates. In particular, we study the existence and stability of the coexistence states, and the semi-trivial equilibria or the unique coexistence state is the global attractor can be established under some suitable conditions. Our mathematical approach is based on Lyapunov–Schmidt reduction, the implicit function theory and spectral theory.

1. Introduction

In this paper, we shall consider the following two competition species in the unstirred chemostat model

$$\begin{cases} S_t = \Delta S - auf(S,k_1) - bvf(S,k_2), & x \in \Omega,\ t > 0, \\ u_t = \Delta u + auf(S,k_1), & x \in \Omega,\ t > 0, \\ v_t = \Delta v + bvf(S,k_2), & x \in \Omega,\ t > 0 \end{cases} \tag{1.1}$$

with boundary conditions

$$\frac{\partial S}{\partial n} + \gamma(x)S = S^0(x), \quad x \in \partial\Omega,\ t > 0,$$
$$\frac{\partial u}{\partial n} + \gamma(x)u = \frac{\partial v}{\partial n} + \gamma(x)v = 0, \quad x \in \partial\Omega,\ t > 0,$$

where Ω is a bounded region in $\mathbb{R}^N (N \geq 1)$ with n is the outer-unit normal vector on $\partial\Omega, S$ is the concentration of nutrient, u and v are the concentration of the competing species, $a, b > 0$ are the maximal

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growth rates, the response function $f(S,k_i)=\frac{S}{k_i+S}$ with $k_i>0$ being the Michaelis–Menten constant, $\gamma(x), S^0(x)$ are continuous on $\partial\Omega$ and $\gamma(x), S^0(x)\geq 0, \not\equiv 0$. Let $\Gamma_1=\{x\in\partial\Omega:\gamma(x)=0\}$, we also assume that $\Gamma_1\neq\emptyset, \Gamma_1\neq\partial\Omega$ and $S^0(x)>0$ on Γ_1.

Let $Z=S+u+v$. Then Z satisfies

$$Z_t=\Delta Z,\quad x\in\Omega,\ t>0,\qquad \frac{\partial Z}{\partial n}+\gamma(x)Z=S^0(x),\quad x\in\partial\Omega,\ t>0.$$

By similar arguments as in Lemma 2.1 of [1], we have $\lim_{t\to\infty}Z(x,t)=z(x)$ uniformly on $\bar{\Omega}$, where $z(x)$ is the unique positive solution to the following problem

$$\Delta z=0,\quad x\in\Omega,\qquad \frac{\partial z}{\partial n}+\gamma(x)z=S^0(x),\quad x\in\partial\Omega.$$

System (1.1) is reduced to the following limiting competition system

$$\begin{cases} u_t=\Delta u+auf(z-u-v,k_1), & x\in\Omega,\ t>0,\\ v_t=\Delta v+bvf(z-u-v,k_2), & x\in\Omega,\ t>0,\\ \dfrac{\partial u}{\partial n}+\gamma(x)u=\dfrac{\partial v}{\partial n}+\gamma(x)v=0, & x\in\partial\Omega, t>0. \end{cases} \tag{1.2}$$

Since only nonnegative solutions are meaningful, we redefine

$$\tilde{f}(S,k)=\begin{cases} f(S,k) & S\geq 0,\\ \tan^{-1}\left(\dfrac{2S}{k}+1\right)-\dfrac{\pi}{4}, & S<0. \end{cases}$$

Obviously, $\tilde{f}(S,k)\in C^1(\bar{\Omega})$, and we still let $f(S,k)$ denote $\tilde{f}(S,k)$.

Let λ_0 be the principal eigenvalue of the following linear problem

$$\Delta\phi+\lambda f(z,k)\phi=0,\quad x\in\Omega,\qquad \frac{\partial\phi}{\partial n}+\gamma(x)\phi=0,\quad x\in\partial\Omega$$

with the corresponding eigenfunction $\phi>0$ on $\bar{\Omega}$.

It is well-known that [2] for any $a>\lambda_0$, the following scalar equation

$$\Delta u+auf(z-u,k)=0,\quad x\in\Omega,\qquad \frac{\partial u}{\partial n}+\gamma(x)u=0,\quad x\in\partial\Omega$$

has a unique positive solution which we denote by θ_a. In particular, $0<\theta_a<z, \lim_{a\to\lambda_{0+}}\theta_a=0$, and $\lim_{a\to+\infty}\theta_a=z$. Moreover, θ_a is non-degenerate and linearly stable.

In [3], So and Waltman considered the existence of coexistence states for the competition organisms in a chemostat in the one dimensional case. In particular, Wu [2] treated b as the bifurcation parameter and gave the global bifurcation structure of the coexistence states of system (1.1). Moreover, Nie and Wu [4] showed that system (1.1) has a unique coexistence state if the maximal growth rates a, b are near the principal eigenvalues and under some suitable conditions. In fact, lots of chemostat models have attracted the attention of both mathematicians and biologists in recent years, and various forms of the competition chemostat systems have been considered in [5,1,6–14] and references therein.

What will happen when the two species are slightly different? In particular, Wu [2] also showed in the special case when $k_1=k_2$, system (1.1) possesses coexistence solutions if and only if $a=b$ when the maximal growth rate is constant, however, we want to know when the maximal growth rate is nonconstant, whether system (1.1) has a coexistence state when the two species are almost identical except for their maximal growth rates. In fact, we will show that the answer can be yes or no, depending on circumstances.

For the sake of convenience, one can apply a change of coordinate $(\bar{S},\bar{u},\bar{v})=(S/k,u/k,v/k)$ to get rid of the parameter since k does not play any role in the analysis, and hence $f(S,k)$ we denote $f(\bar{S},1)$. Moreover, we drop the bars of $\bar{S},\bar{u}$ and $\bar{v}$, and we shall denote $f(\bar{S},1)$ by $f(S)$.