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Exact internal controllability of nodal profile for first order quasilinear hyperbolic systems☆

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ABSTRACT

For 1-D first order quasilinear hyperbolic systems without zero eigenvalues, based on the theory of semi-global C^1 solution, by means of the results on both the exact boundary controllability and the exact boundary controllability of nodal profile, using an extension method, the exact controllability of nodal profile can be obtained purely with the help of internal controls. Moreover, using a perturbation method, by adding suitable internal controls to the part of equations corresponding to zero eigenvalues, the corresponding exact internal controllability of nodal profile for 1-D first order quasilinear hyperbolic systems with zero eigenvalues can be also obtained.

1. Introduction

There is a complete theory on the local exact boundary controllability for general 1-D first order quasilinear hyperbolic systems. Li Tatsien et al. (see [6,9–12]) studied the exact boundary controllability for general 1-D first order quasilinear hyperbolic systems, and the related conclusions were generalized to the case on a tree-like network (see [3,6]). J.M. Coron et al. (see [2]) studied the boundary feedback control for Saint-Venant equations in networks, according to the character of the zero eigenvalues around the equilibrium state, established the corresponding exact boundary controllability (see [1]).

In pipeline network, the gas flow is controlled through the compressors to provide the desired amount of gas to the consumers, which leads to the study on the exact boundary controllability of nodal profile. M. Gugat et al. (see [5]) first proposed the problem of the nodal profile control, while, Li Tatsien (see [7]) precisely gave the conception of the exact boundary controllability of nodal profile, and established the theory of the exact boundary controllability of nodal profile for general first order quasilinear hyperbolic systems soon after, and then generalized this theory to the case on a tree-like network (see [4,13]).

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Internal controls are frequently used in the control problem for wave equations (see [17]). For 1-D first order quasilinear hyperbolic systems with general nonlinear boundary conditions, based on the theory of the semi-global C^1 solution, using an extension method, the exact controllability can be obtained in a shorter time with the help of additional internal controls. For some special but meaningful equations, the exact controllability can be obtained purely with the help of internal controls (see [16]). In [14], using the combination of boundary controls and internal controls, the exact controllability can be obtained in a shorter time. The goal in this paper is to improve the results of [14], such that, without boundary controls, the exact controllability of nodal profile can be obtained purely by internal controls.

Consider the following 1-D first order quasilinear hyperbolic system

$$\frac{\partial u}{\partial t} + A(u)\frac{\partial u}{\partial x} = F(u), \tag{1.1}$$

where $u = (u_1, \dots, u_n)^T$ is the unknown vector function of (t, x), $A(u)$ is a given $n \times n$ matrix with suitably smooth elements $a_{ij}(u)(i, j = 1, \dots, n)$, $F(u) = (f_1(u), \dots, f_n(u))^T$ is a smooth vector function satisfying

$$F(0) = 0. \tag{1.2}$$

By (1.2), $u = 0$ is an equilibrium of system (1.1).

By hyperbolicity, on the domain under consideration, the matrix $A(u)$ has n real eigenvalues $\lambda_i(u)(i = 1, \dots, n)$ and a complete set of left eigenvectors $l_i(u) = (l_{i1}(u), \dots, l_{in}(u))(i = 1, \dots, n)$:

$$l_i(u)A(u) = \lambda_i(u)l_i(u) \tag{1.3}$$

with

$$\det|l_{ij}(u)| \neq 0. \tag{1.4}$$

System (1.1) can be equivalently rewritten into the characteristic form:

$$l_i(u)(\frac{\partial u}{\partial t} + \lambda_i(u)\frac{\partial u}{\partial x}) = \widetilde{F}_i(u) \overset{def.}{=} l_i(u)F(u) \quad (i = 1, \dots, n), \tag{1.5}$$

in which the ith equation consists of only the directional derivative of the unknown function u with respect to t along the ith characteristic direction $\frac{dx}{dt} = \lambda_i(u)$. Obviously,

$$\widetilde{F}_i(0) = 0 \quad (i = 1, \dots, n). \tag{1.6}$$

Adding suitable internal controls $c_i(t, x)(i = 1, \dots, n)$ to system (1.5), we have

$$l_i(u)(\frac{\partial u}{\partial t} + \lambda_i(u)\frac{\partial u}{\partial x}) = \widetilde{F}_i(u) + c_i(t, x) \quad (i = 1, \dots, n), \tag{1.7}$$

where

$$c_i(t, x) = l_i(\vartheta)(\frac{\partial \vartheta}{\partial t} + \lambda_i(\vartheta)\frac{\partial \vartheta}{\partial x}) - \widetilde{F}_i(\vartheta) \quad (i = 1, \dots, n), \tag{1.8}$$

in which $\vartheta = \vartheta(t, x)$ is a C^1 vector function of (t, x), while the supports of the internal controls $c_i(t, x)(i = 1, \dots, n)$ are related to the distribution of the eigenvalues and the location of the nodal profile.

This paper is organized as follows: in Section 2, we briefly recall the previous results on the exact boundary controllability and the exact boundary controllability of nodal profile for general 1-D first order quasilinear hyperbolic systems without zero eigenvalues, and the C^1 extension of the solution. Then, using an extension method, the exact controllability of nodal profile can be obtained purely with the help of internal controls in Section 3. Moreover, by adding suitable internal controls to the part of equations corresponding to zero eigenvalues, the exact internal controllability of nodal profile for general 1-D first order quasilinear hyperbolic systems with zero eigenvalues can be obtained in Section 4.