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Existence of global bounded classical solution to a quasilinear attraction–repulsion chemotaxis system with logistic source

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ABSTRACT

This paper concerns the global existence of bounded classical solution to a quasilinear attraction–repulsion chemotaxis system with logistic source

$$\begin{cases} u_t = \nabla\cdot(D(u)\nabla u) - \chi\nabla\cdot(u\nabla v) + \xi\nabla\cdot(u\nabla w) + \kappa u - \mu u^2 & \text{in } \Omega, \\ v_t = \Delta v + \alpha u - \beta v & \text{in } \Omega, \\ w_t = \Delta w + \gamma u - \delta w & \text{in } \Omega, \end{cases}$$

under homogeneous Neumann boundary condition, with positive parameters $\chi, \xi, \kappa, \mu, \alpha, \beta, \gamma, \delta$ and $D(s) \geq c_0 s^{m-1}$ for $s > 0$, $D(0) > 0$. We prove that in dimension three there exists a unique global bounded classical solution provided that $m > \frac{6}{5}$ and $\mu > 0$. With an additional assumption $D(s) \leq C_0(s^{m-1}+1)$ for $s > 0$, we prove that for any $m \in \left(1, \frac{6}{5}\right]$, there exists a constant $\mu_0 = \mu_0(m)$ such that for all $\mu > \mu_0$ the above problem admits a unique global bounded classical solution.

1. Introduction

In this paper, we study the existence of global bounded classical solution[1] to the following dimensionless attraction–repulsion chemotaxis system

$$\begin{cases} u_t = \nabla\cdot(D(u)\nabla u) - \chi\nabla\cdot(u\nabla v) + \xi\nabla\cdot(u\nabla w) + \kappa u - \mu u^2 & \text{in } \Omega, \\ \rho v_t = \Delta v + \alpha u - \beta v & \text{in } \Omega, \\ \tau w_t = \Delta w + \gamma u - \delta w & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = \dfrac{\partial w}{\partial \nu} = 0 & \text{on } \partial\Omega, \\ (u,v,w)|_{t=0} = (u_0(x), v_0(x), w_0(x)) & \text{in } \Omega, \end{cases} \tag{1.1}$$

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[1] We say (u, v, w) is a classical solution to (1.1) in $(0,T)\times\Omega$, provided that $(u,v,w) \in [C^0(\bar{\Omega}\times[0,T)) \cup C^{2,1}(\bar{\Omega}\times(0,T))]^3$ and satisfies (1.1) pointwise.

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where $\Omega \subset \mathbb{R}^3$ is a bounded smooth domain, the scalar function $u = u(x,t)$ denotes the cell density, $v = v(x,t)$ and $w = w(x,t)$ denote the concentration of an attractive signal and the concentration of a repulsive signal, respectively. The logistic source term $\kappa u - \mu u^2$ models proliferation and death of cells which intend to prevent an unlimited growth of the cell density. The initial datum $u_0(x)$, $v_0(x)$ and $w_0(x)$ are all nonnegative and belong to $W^{1,\infty}(\Omega)$, the parameters $\chi, \xi, \kappa, \alpha, \beta, \gamma, \delta$ are all assumed to be positive, $\rho = 0, 1$ and $\tau = 0, 1$ are constants. The diffusion coefficient function D satisfies

$$D \in C^2([0, +\infty)), \qquad D(0) > 0, \tag{1.2}$$

and

$$D(s) \geq c_0 s^{m-1} \quad \text{for } s > 0, \tag{1.3}$$

with a constant $c_0 > 0$.

System (1.1) is a generalized model of the classical Keller–Segel model [18] which describes a biological process where cells interact with a combination of repulsive and attractive signal chemicals (cf. [29,34]). The attraction–repulsion chemotaxis model (1.1) with $\rho = \tau = 1$ and $\kappa = \mu = 0$ was proposed in [29] to describe the aggregation of microglia observed in Alzheimer's disease and in [34] to address the quorum effect in the chemotactic process. Since the chemical molecules are much smaller than cells in size, in general chemicals diffuse much faster than cells. Therefore, system (1.1) can be approximated by setting $\rho = 0$ and/or $\tau = 0$. We call such approximation quasi-steady-state approximation.

When $\xi = \kappa = \mu = 0$, the equation of w is decoupled from the system (1.1) and the system of u and v becomes

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u) - \chi \nabla \cdot (u \nabla v), & x \in \Omega, t > 0 \\ \rho v_t = \Delta v + \alpha u - \beta v, & x \in \Omega, t > 0. \end{cases} \tag{1.4}$$

When $D(s) \equiv 1$ and $\rho = 1$, system (1.4) becomes the classical parabolic–parabolic Keller–Segel system, such system has been extensively studied, see for example [10–13,16,30–32,35,42–44] and references therein. For the study of the parabolic-elliptic Keller–Segel system of quasilinear type, namely, $\rho = 0$ with general $D(s)$ in (1.4), we refer to [2,3,5,6,8,9,19] and references therein. For the study of the parabolic–parabolic Keller–Segel system of quasilinear type, namely, $\rho = 1$ with general $D(s)$ in (1.4), we refer to [4,7,14,15,21] and references therein.

When $D(s) \equiv 1$ and $\kappa = \mu = 0$, namely, the logistic term vanishes, system (1.1) is well studied. We mention some of the results here. When $\rho = \tau = 0$ or $\rho = \tau = 1$, a global unique classical solution to (1.1) exists if the repulsion dominates over attraction, i.e., $\chi\alpha - \xi\gamma < 0$, see [17,27,28,37]. When $\rho = \tau = 1$ and the repulsive cancels the attraction, i.e., $\chi\alpha - \xi\gamma = 0$, Lin, Mu and Wang [26] showed the global existence of classical solution to (1.1). When $\rho = \tau = 0$ and the attraction dominates over repulsion, i.e., $\chi\alpha - \xi\gamma > 0$, Tao and Wang [37] established a blow up result for dimension two. In [24], Lin and Mu established the existence of a unique global bounded classical solution in two dimension provided that $\rho = \tau = 1$ and that the initial data u_0 is small in $L^1(\Omega)$ norm.

When $D(s) \equiv 1$, $\kappa \neq 0$ and $\mu \neq 0$, system (1.1) was studied by Li and Xiang [23]. They considered the parabolic–parabolic system (i.e., $\rho = \tau = 1$) in one and two dimensional space and the parabolic-elliptic system (i.e., $\rho = \tau = 0$) in higher dimensional space. For the former case, they proved the existence of unique global bounded classical solution.

For general D, and $\kappa = \mu = 0$, Lin, Mu and Gao [25] studied system (1.1) in the case that $\rho = 1$ and $\tau = 0$. More precisely, they proved the existence of a unique bounded and global classical solution provided that $D(s) > 0$ for all $s \geq 0$ and that

$$D \geq D_0 s^{-\theta} \quad \forall\, s > 0, \text{ where } \theta < \frac{2}{n} - 1.$$