Nonlinear Analysis 164 (2017) 100–117

Contents lists available at ScienceDirect

Nonlinear Analysis

www.elsevier.com/locate/na

Existence and multiplicity of solutions for fractional Choquard equations

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ARTICLE INFO

Article history:
Received 27 January 2017
Accepted 26 July 2017
Communicated by Enzo Mitidieri

Keywords:
Critical nonlinearity
Ground state solutions
Fractional Choquard equation
Variational method
Lusternik–Schnirelmann category theory

ABSTRACT

We consider the following fractional order Choquard equation

$$(-\triangle)^{\alpha/2}u(x) + (\lambda V(x) - \beta)u = (|x|^{-\mu} * |u|^{2^*_\mu})|u|^{2^*_\mu - 2}u,\ x \in R^n,$$

with the nonlinearity in the critical growth, where $\alpha \in (0,2)$, $n \geq 3$, $\lambda, \beta \in R^+$ and $2^*_\mu = (2n-\mu)/(n-\alpha)$. Using the variational method, we establish the existence and multiplicity of weak solutions.

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1. Introduction

In this paper, we consider the following fractional Choquard equation

$$\begin{cases} (-\triangle)^{\alpha/2}u + (\lambda V(x) - \beta)u = (|x|^{-\mu} * |u|^p)|u|^{p-2}u,\ x \in R^n, \\ u \in H^{\alpha/2}(R^n), \end{cases} \tag{1}$$

in the critical case $p = 2^*_\mu = \frac{2n-\mu}{n-\alpha}$, where $\alpha \in (0,2)$ and $\mu \in (0,n)$.

The fractional Laplacian in R^n is a nonlocal pseudo-differential operator taking the form

$$\begin{aligned} (-\triangle)^{\alpha/2}u(x) &= C_{n,\alpha} PV \int_{R^n} \frac{u(x)-u(y)}{|x-y|^{n+\alpha}} dy \\ &= C_{n,\alpha} \lim_{\varepsilon\to 0} \int_{R^n \setminus B_\varepsilon(x)} \frac{u(x)-u(y)}{|x-y|^{n+\alpha}} dy, \end{aligned} \tag{2}$$

where $C_{n,\alpha}$ is a normalization constant and PV is the Cauchy principal value. In this paper, we consider the fractional Laplacian in the weak sense.

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http://dx.doi.org/10.1016/j.na.2017.07.011
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We denote by $H^{\alpha/2}(R^n)$ the homogeneous fractional space. It is defined as the completion of $C_0^\infty(R^n)$ that

$$H^{\alpha/2}(R^n) = \left\{ u \in L^2(R^n) \mid \int_{R^n}\int_{R^n} \frac{(u(x)-u(y))^2}{|x-y|^{n+\alpha}} dxdy < \infty \right\}.$$

This space is endowed with the norm

$$\|u\|_{H^{\alpha/2}(R^n)} = \left(\int_{R^n}\int_{R^n} \frac{(u(x)-u(y))^2}{|x-y|^{n+\alpha}} dxdy + \int_{R^n} |u|^2 dx \right)^{\frac{1}{2}}.$$

Let $H_0^{\alpha/2}(\Omega)$ be the completion of $C_0^\infty(\Omega)$ under the norm $\|\cdot\|_{H^{\alpha/2}}$. It is endowed with the norm defined as

$$\|u\|_{H_0^{\alpha/2}(\Omega)} = \int_{\mathcal{Q}} \frac{|u(x)-u(y)|^2}{|x-y|^{n+\alpha}} dxdy,$$

where $\mathcal{Q} := (R^n \times R^n) \setminus \mathcal{O}$ and $\mathcal{O} := C\Omega \times C\Omega \subset R^n \times R^n$. The homogeneous fractional Sobolev space $D^{\frac{\alpha}{2},2}(R^n)$ is defined as the completion of $C_0^\infty(R^n)$

$$D^{\alpha/2,2}(R^n) = \left\{ u \in L^2(R^n) \mid \frac{|u(x)-u(y)|}{|x-y|^{\frac{n+\alpha}{2}}} \in L^2(R^n) \right\},$$

and it is endowed with the norm

$$\|u\|^2_{D^{\alpha/2,2}(R^n)} = \int_{R^{2n}} \frac{|u(x)-u(y)|^2}{|x-y|^{n+\alpha}} dxdy.$$

In recent years, the fractional Laplacian has attracted much attention. It appears in diverse physical phenomena, such as anomalous diffusion and quasi-geostrophic flows, turbulence and water waves, molecular dynamics, and relativistic quantum mechanics of stars. It also has various applications in probability and finance. In particular, the fractional Laplacian can be understood as the infinitesimal generator of a stable Lévy diffusion process and appear in anomalous diffusions in plasmas, flames propagation and chemical reactions in liquids, population dynamics, geographical fluid dynamics, and American options in finance. For readers who are interested in the applications of the fractional Laplacian, please refer to [1,4] and the references therein.

When $V(x) = 1$, $\beta = 0$ and $\frac{2n-\mu}{n} < p < \frac{2n-\mu}{n-\alpha}$, (1) becomes the following nonlocal problem

$$(-\triangle)^{\alpha/2}u(x) + \lambda u = (|x|^{-\mu} * |u|^p)|u|^{p-2}u.$$

In [8], the authors obtained regularity of weak solutions, existence and properties of ground states, as well as multiplicity and nonexistence of solutions.

When $\alpha = 1$, the above problem has been used to model the dynamics of pseudo-relativistic boson stars. In particular, when $\lambda = 1$, $\mu = 1$ and $p = 2$, (1) deduces to

$$\sqrt{(-\triangle)}u + u = (|x|^{-1} * |u|^2)u,$$

and in [10], Frank and Lenzmann proved analyticity and radial symmetry of ground state solutions. Moreover, in [9], it was shown that the dynamical evolution of boson stars is effectively described by the nonlinear evolution equation with mass parameter $m \geq 0$

$$i\partial_t\psi = \sqrt{-\triangle + m^2}\psi - (|x|^{-1} * |\psi|^2)\psi$$

for the wave field $\psi : [0, T] \times R^3 \to \mathbb{C}$. In fact, this dispersive nonlinear L_2 critical PDE displays a rich variety of phenomena such as stable or unstable traveling solitary waves and finite-time blowup.

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